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ELEMENTARY

STATICS AND DYNAMICS,

DESIGNED FOR THE USE OF SCHOOLS,

AND OF

CANDIDATES FOR SECOND-CLASS CERTIFICATES,

WITH NUMEROUS EXERCISES.

BY

WILLIAM H. IRVINE, B.A.

MATHEMATICAL MASTER, COLLEGIATE INSTITUTE, KINGSTON.

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PREFACE.

The following elementary work is intended to cover the course prescribed by the Education Department for Second Class teachers.

The demonstrations have been carefully made so as to be easily intelligible to those who do not understand Trigonometry or the higher Mathematics, but have a fair knowledge of some of the elementary problems in Euclid and Algebra.

In a few instances where I thought mathematical demonstration too difficult for the pupil, other proof has been substituted, or it has been left for the teacher to explain.

Examples in illustration are given on all the points which were thought to present most difficulty to a beginner; and numerous examples are given for exercise, to test the student's knowledge of the subject.

I shall be thankful to have any errors or omissions in the text pointed out.

W. H. I.

September, 1886.

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STATICS.

CHAPTER I.

1. **Force.**—Any cause which produces or tends to produce motion, or change of motion, in a body is called a force.

2. **Equilibrium.**—When two or more forces so act on a body that no motion ensues; the forces are said to be in equilibrium.

3. **Statics** is the science that investigates the relations existing among forces in equilibrium.

4. **Matter.**—The material or substance composing a body is termed its matter.

5. **Mass.**—The quantity of matter existing in a body is called its mass. It must be carefully distinguished from weight.

6. **Momentum** is the product of the mass into the velocity.

7. **Measurement of Force.**—A force may be measured by the momentum it produces, or by the weight it will sustain.

8. **Force** may be exerted in three different ways, viz., by

(1) **Pressure.**—By pressure we mean such forces as are prevented from producing motion.

(2) **Tension.**—Force transmitted by means of a cord or rod used as a string is called the tension of the cord or rod.

(3) **Attraction.**—Such as the attraction of gravity and magnetic attraction.

9. **Weight** is the pressure which anybody at rest exerts towards the earth.

10. **Forces** may be represented by solids, superficies, or lines. If we represent a force of 1 lb. by a sphere of 1 inch in diameter, or by the area of a square whose side is 6 inches, or by a line 9 inches long, then a force of 64 lbs. might be represented by a sphere 4 inches in diameter, or by a square whose side is 48 inches or by a line 576 inches long. But as forces may differ from each other not only in magnitude but also in direction and point of application, the straight line is best suited to represent a force; as it may be drawn so as to represent all three elements, viz: (1) point of application, (2) direction, and (3) magnitude, at the same time.

11. **Transmissibility of Force.**—A force acting upon a body may be supposed to act at any point of the body in its line of direction.

EXERCISE I.

1. Define Force. In how many ways may forces be exerted? Give examples of each.

2. If a force of 4 lbs. be represented by a line 7 inches long, find length of line that will represent a force of (*a*) 6 lbs., (*b*) 12 lbs., (*c*) 18 lbs.

3. If a line whose length is 2 ft. 3 in. represent a force of 54 lbs., find the force represented by a line (*a*) 10 in., (*b*) 1 ft. 9 in., (*c*) 5 ft. in length.

4. Draw lines to represent forces of 5 oz., 2 lbs., 9 lbs., and 15 lbs. respectively, taking unit of length to represent $2\frac{1}{2}$ oz.

CHAPTER II.

FORCES IN A STRAIGHT LINE.

12. **Resultant**.—The single force which would produce the same effect as that produced by two or more forces, is called their resultant. The several forces are called component forces, when compared with the resultant.

13. **Composition of Forces** is the method of finding the resultant of two or more forces.

14.—**Resolution of Forces** is the method of finding the components of the resultant, or of any single force.

15.—The **resultant** of two or more forces acting on a particle along a straight line, is equal to their algebraical sum.

(NOTE.—If forces acting in one direction are called positive, those acting in a contrary direction must be termed negative.)

EXERCISE II.

1. Find the resultant of two forces of 5 lbs. and 9 lbs. acting on a particle in the same direction.

2. Find the resultant of two forces of 12 lbs. and 8 lbs. acting upon a particle in opposite directions.

3. Under what circumstances is the resultant of two forces the greatest? The least? When is it zero?

4. The resultant of two forces acting along a straight line is 7 lbs., one of the components is also 7 lbs; find magnitude and direction of the other.

5. A, B, C, D & E are points in a straight line, such that AB=5 in., BC=6 in., CD=7 in., and DE=8 in.; forces represented by AB, CB, CD, CE, and DB act along the line; find magnitude of resultant when force AB=10 lbs.

6. Resolve a force of 15 lbs. into two forces whose ratio is as 1 : 2.

7. Resolve a force of 16 lbs. into two forces which shall be to one another as 3 : 4, one of the components being 48 lbs.

8. Weights of 3, 5, 7, and 8 lbs. are suspended by a string; find its tension.

9. Two men pull on a rope in contrary directions with a force of 50 lbs. each; find the tension of the rope.

10. What would the tension of the rope be, in Question 9, if the men pulled with forces of 60 and 80 lbs. respectively.

11. Find resultant in Questions 9 and 10 respectively.

12. A certain cord is capable of sustaining a tension of 20 lbs.; apply two forces of 14 and 18 lbs. to it so as not to break the cord.

13. The greatest resultant of two forces acting on a particle is 50 lbs., their least is 10 lbs.; find the forces.

CHAPTER III.

PARALLELOGRAM OF FORCES.

16. If two forces acting on a particle be represented in magnitude and direction by the two sides of a parallelogram, their resultant will be represented in magnitude and direction by the diagonal of the parallelogram passing through the particle.

EXERCISE III.

ON FORCES ACTING AT RIGHT ANGLES.

1. Two forces of 5 and 12 lbs. respectively act upon a particle at right angles to each other; find the magnitude and direction of their resultant.

> Draw AB to represent the force of 5 lbs. and AC at right angles to represent 12 lbs. Complete the parallelogram ABDC; join AD. Then AD shall represent the resultant both in magnitude and direction.

$AD^2 = AB^2 + BD^2$ (Euc. I., 47), &

$AC = BD$.

$\therefore\ AD^2 = 5^2 + 12^2 = 169$.

$\therefore\ AD = 13$ lbs.

2. The resultant of two forces acting on a particle at right angles to one another is 50 lbs.; find the forces if their ratio is as 7 : 24.

> In fig. 1 AD = resultant = 50 lbs., AB = 7 units, when AC = 24 of these units, which we will call x.

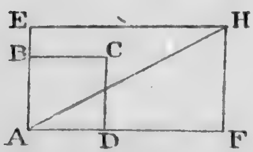
$$AD^2 = AB^2 + BD^2.$$
$$50^2 = (7\,x)^2 + (24\,x)^2.$$

$$2{,}500 = 625\,x^2.$$
$$\therefore\ x = 2. \quad \therefore\ AB = 14 \text{ and } AC = 48,$$
$\therefore$ the fcs. are 14 lbs. and 48 lbs.

Or if $AB = 7$ and $AC = 24$, then $AD = 25$, and fcs. must be in that ratio. $\therefore\ 25 = 50$ lbs.
$\therefore\ 7 = 14$ lbs and $24 = 48$ lbs.

3. Two forces of 6 lbs. and 8 lbs. act along the adjacent sides of a square; find the magnitude and direction of the third force that will produce equilibrium.

Let ABCD = square; produce AB to any point E, and let AE represent 6 lbs., and take AF equal to 8 lbs. on the same scale; complete parallelogram.



Then AH will represent the resultant of AE & AF,

$\therefore$ HA will = 3rd force that will produce equil.

$$HA^2 = AE^2 + EH^2.$$
$$= 6^2 + 8^2.$$
$$= 100.$$
$$\therefore\ HA = 10 \text{ lbs.}$$

4. Two forces of 11 lbs. and 60 lbs. act upon a particle at right angles to each other; find their resultant.

5. The resultant of two fcs. acting at right angles is 85 lbs.: one of the components is 13 lbs.; find the other.

6. Two fcs. whose magnitudes are as 8 : 15 act along the adjacent sides of a rectangle; find the magnitude and direction of the third force that will produce equilibrium.

7. Which will pull the greatest weight, 4 men pulling together each with a force of $3\,x$, or 3 men pulling at right angles to 4 men, each with a force of $2\,x$?

8. A weight is supported by two cords at right angles to each other; if the tensions of the cords are 12 lbs. and 35 lbs. respectively; find the weight.

9. Two fcs. acting along the adjacent sides of a square are to one another as 15 : 112; their resultant is 339 lbs.; find the magnitude of the components.

10. The sum of two fcs. acting at right angles and their resultant is 56 lbs.; one of components is 7 lbs., find other component and resultant.

CHAPTER IV.

FORCES ACTING AT ANGLES OF 30°, 45°, 60°, 90°, 120°, 135° & 150°.

17. In order to find the resultant of fcs. acting at any angle not a right angle, it is necessary to find the relation existing among the sides of a triangle having the given angle. This can only be done for a few angles, without the aid of trigonometry; the simpler of which are the above angles.

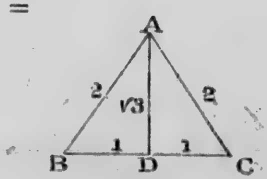
I. Let ABC be an equilateral triangle, and $\therefore$ each angle $= 60°$. From A let fall perpendicular AD; it bisects the base and vertical angle, $\therefore$ BD $=$ DC and angle BAD $= 30°$,

Let $BD = 1$, then $BC = 2 = AC = AB$.

$AB^2 - BD^2 = AD^2$.

i.e., $4 - 1 = AD^2$.

$\therefore AD = \sqrt{3}$.

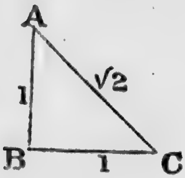


Therefore in any right-angled triangle whose acute angles are 30° and 60° respectively, if the side oppo-

site 30° be represented by 1, the side opposite 90° will be represented by 2, and the side opposite 60° by $\sqrt{3}$.

If BD $= x$, AD $= x\sqrt{3}$, and AB $= 2x$.

II. Let ABC be a right angled triangle having right angle ABC, and angle at A 45° and $\therefore \angle$ at C = 45°, $\therefore$ BA = BC.



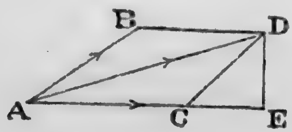
Let AB = 1 = BC.

$$AC^2 = AB^2 + BC^2.$$
$$= 1 + 1 = 2.$$
$$\therefore AC = \sqrt{2}.$$

$\therefore$ if the side opposite 45° in a right angled triangle be represented by 1 the side opposite 90° will be represented by $\sqrt{2}$.

EXAMPLES.

1. To find the resultant when fcs. of 8 and 10 lbs. act upon a point at an angle of 30°.



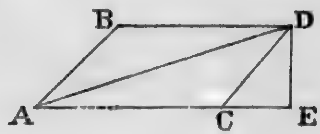
Let AB = 8, AC = 10 and $\angle$ BAC = 30°; to find AD. Let fall $\perp$ DE meeting AC produced in E.

Then CD = AB = 8 and $\angle$ DCE = BAC = 30°, and DEC = 90° $\therefore$ CDE = 60°;

$\because$ CD = 8, $\therefore$ DE = 4, and CE = $4\sqrt{3}$, $\therefore$ AE = $10 + 4\sqrt{3}$.

$$AD^2 = AE^2 + DE^2.$$
$$= (10 + 4\sqrt{3})^2 + 4^2.$$
$$= 100 + 48 + 80\sqrt{3} + 16 = 164 + 80\sqrt{3}.$$
$$\therefore AD = \sqrt{164 + 80\sqrt{3}}.$$

2. To find the resultant when fcs. of 10 and 18 lbs. act upon a point at an angle of 60°.



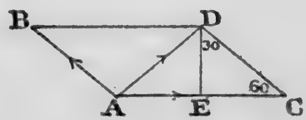
Let $AB = 10$, $AC = 18$ and $\angle BAC = 60°$; to find AD. Let fall $\perp$ DE. Then $\angle DCE = 60°$, and $DEC = 90°$, $\therefore CDE = 30°$,

$\therefore$ sides must be in ratio of 1, 2 and $\sqrt{3}$,

$\therefore$ if $CD = 10$, $CE = 5$ and $DE = 5\sqrt{3}$.

$$AD^2 = AE^2 + DE^2.$$
$$= (18 + 5)^2 + (5\sqrt{3})^2.$$
$$= 529 + 75 = 604.$$
$$\therefore AD = \sqrt{604} = 24.57 \text{ lbs.}$$

3. To find the resultant when fcs. of 20 and 30 lbs. act upon a particle at an angle of 120°.



Let $AB = 20$, $AC = 30$ lbs. and $\angle BAC = 120°$.

$\therefore ACD = 60°$ (Euc. I., 29) and $DEC = 90°$, $\therefore EDC = 30°$ and $DC = 20$, $\therefore EC = 10$ and $DE = 10\sqrt{3}$.

$$AD^2 = AE^2 + DE^2.$$
$$= (30 - 10)^2 + (10\sqrt{3})^2.$$
$$= 400 + 300 = 700.$$
$$\therefore AD = \sqrt{700} = 10\sqrt{7} \text{ lbs.}$$

EXERCISE IV.

1. Given right angled triangle having an angle of 60° to find the sides when the hypothenuse is 10 ft. in length.

2. ABC is a triangle right angled at C, the angle $BAC = 30°$; find sides AB and BC, when AC is (*a*) 6 ft., (*b*) 8 ft., (*c*) $4\sqrt{3}$ ft., (*d*) $\frac{2}{\sqrt{3}}$ ft.

3. ABC is a triangle having the angle BAC $= 120°$; find side BC when (*a*) AC is 10 ft. and BA 4 ft., (*b*) AC 5 ft. and BA 6 ft.

4. Given a right angled isos. triangle having hypothenuse 12 ft. in length; find the sides.

5. Given ABC a triangle having angle BAC $= 150°$; find length of BC, when AC is 18 ft. and AB is 6 ft.

6. The sides BA, AC of the triangle ABC are $3a$ and $4a$ ft. respectively; find BC if angle BAC = (*a*) 135°, (*b*) 30°, (*c*) 45°.

7. If in triangle ABC, AB $= 2$, BC $= 2\sqrt{3}$, and AC $= 4$; find the angles

If BD the $\perp$ on AC $= \sqrt{3}$. Prove that DBC $= 60°$ and DBA$=30°$.

EXERCISE V.

PARALLELOGRAM OF FORCES.

1. Two fcs. each of 50 lbs. act upon a point at an angle of 60°; find the resultant.

2. Forces of 12 and 20 lbs. respectively act upon a point at an angle of 60°; find the resultant.

3. Forces of 8 and 10 lbs. respectively act upon a particle at an angle of 30°; find the resultant.

4. Two fcs. each of 6 lbs. act upon a point at an angle of 45°; find the resultant.

5. Forces of 10 and $12\sqrt{3}$ lbs. respectively act upon a point at an angle of 150°; find the resultant.

6. Forces of 7 and 33 lbs. respectively have a resultant of 37 lbs.; prove that the angle between the fcs. is 60°.

7. Two fcs. of 15 and 40 lbs. respectively act upon a point at an angle of 120° ; find their resultant.

8. Two fcs. acting at angle of 135° have a resultant of 10 $\sqrt{5}$ lbs., one of the fcs. is 30 lbs., find the other.

9. Two fcs. of 6 and 8 $\sqrt{3}$ acting at a point have a resultant of 2 $\sqrt{93}$ lbs. ; find the angle between the fcs.

10. Two fcs. of 30 and 35 $\sqrt{2}$ lbs. are kept in equilibrium by a force of 5 $\sqrt{218}$; find the angle between the fcs.

11. Two equal fcs. act at an angle of 120° ; their resultant is 10 lbs. ; find the fcs.

12. ABDC is a rhombus, having the angle at A = 60° ; AD, CB intersect in E ; fcs. act along AB, AD, AE and BE, and are proportioned to them ; find magnitude of their resultant, if the force AB is 12 lbs.

13. If a force of 60 lbs. be resolved into two equal fcs. acting at an angle of 30°, what is the magnitude of either component ?

14. The resultant of two equal fcs. acting at an angle of 135° is 20 lbs. ; find the components.

15. The resultant of two fcs. in the ratio of 2 : 3, acting at an angle of 150° is 24 lbs. ; find the fcs.

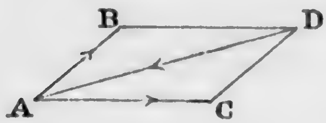
16. The resultant of two fcs. acting at an angle of 60° is 21 lbs., one of the components is 9 lbs. ; find the other.

CHAPTER V.

TRIANGLE OF FORCES.

18. If three forces acting upon a point be in equilibrium, and if any triangle be drawn whose sides are

severally parallel, or severally make the same angle with their directions, the fcs. will be to one another as the sides of the triangle; and conversely if the three fcs. be as the sides of the triangle they will be in equilibrium.



Let AB, AC, and DA represent three fcs. in equilibrium acting at the point A. Complete the parallelogram ABDC. AB and AC have a resultant AD, ∴ DA will keep it in equilibrium. Therefore the lines AB, AC, and DA represent the fcs. in magnitude and direction. But BD is equal and parallel to AC, ∴ AB, BD, and DA represent the fcs. in magnitude and direction.

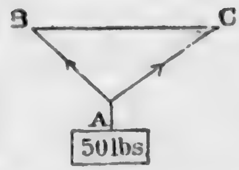
NOTE.—Not in point of application. The sides of the triangle, ABD, will remain the same no matter in what position it be placed, ∴ the sides will represent the magnitude of the fcs. so long as they severally make the same angle with the direction of the fcs.

EXERCISE VI.

1. Can three fcs. represented in magnitude by 3, 4, and 8 lbs. respectively, keep a particle at rest?

   8 is greater than 3 + 4, ∴ lines whose lengths are proportionate to 8, 3, and 4, would not form a triangle, ∴ the fcs. could not act so as to be in equilibrium.

2. A weight of 50 lbs. is sustained by two cords whose lengths are 15 and 20 inches respectively, fastened to two points lying in the same horizontal line. Find the tensions in the cords when they are at right angles to each other.

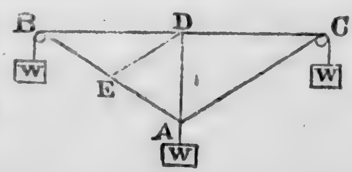


Let AB = 15, AC = 20, and ∠ BAC = 90° ∴ BC = 25.

BC is at right angles to direction of weight, AC at right to tension of cord AB, and AB at right angles to direction of tension in cord AC. The sides of the triangle ABC severally make the same angle with the direction of the fces. ∴ its sides must be proportional to the fcs., *i.e.*, BC = weight, when BA = tension in cord AC, and CA = tension in cord AB.

BC = 25 = 50 lbs.
CA = 20 = 40 lbs. tension in cord AB,
and AB = 15 = 30 lbs. " " AC.

3. A cord having equal weights attached to its extremities passes over pulleys placed at B and C in the same horizontal line 60 in. apart, and through a smooth ring A from which a weight of 160 lbs. is attached; what must be the magnitude of the equal weights, that A may rest exactly 16 inches below the line BC.



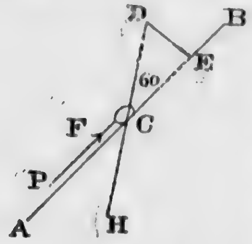
Let the vertical through A meet BC in D,

∵ ABC is an isos. triangle, and AD is ⊥ to BC.

∴ D is the middle point of BC. Draw DE // AC, then E is the middle point of BA. The triangle ADE has its sides parallel to the direction of the fcs., and ∴ must represent the fcs. in magnitude.

AD = 16, BD = $\frac{1}{2}$ BC = 30 ∴ BA = 34 (Euc. I. 47).
AE = $\frac{1}{2}$ BA = 17, and DE = $\frac{1}{2}$ AC = 17.
DA = 16 = 160 lbs.
AE = DE = 17 = 170 lbs. = tension in each cord, ∴ the equal weights = 170 lbs. each.

4. If a weight of 20 lbs. slides along a smooth rod inclined at an angle of 60° with the vertical line, what force acting along the rod will sustain the weight, and what is the pressure on the rod ?



Let AB represent the rod. The three forces are the weight acting vertically down, the force P acting along the rod, and the reaction of the rod (which is always at right angles to the surface in contact). Let fall $\perp$ DE from any point in direction of weight. Then $\because$ DCE = 60 $\therefore$ CDE = 30°

$\therefore$ if DC = 2, CE = 1, and DE = $\sqrt{3}$.

2 = 20.

1 = 10 and $\sqrt{3} = 10\sqrt{3}$ = reaction.

5. Can a particle be kept at rest by three fcs. whose magnitude are as 4, 5 and 8 lbs respectively ?

6. Shew how to keep a particle at rest by means of three equal forces.

7. Shew that if three fcs. are in equillibrium any one of them must be greater than the difference between the other two.

8. Three fcs. acting at a point are in equillibrium, the greatest is 13 lbs. and the least is 5 lbs., and the angle between two of the forces is a right angle, find the other force.

9. Three fcs. represented by 2, 2 and 4 lbs. act upon a particle in directions parallel to the sides of an equilateral triangle taken in order ; find their resultant.

10. If fcs. of 9 lbs. and 40 lbs. have a resultant of 41 lbs., at what angle do they act ?

11. Two fcs. whose magnitude are as 8 : 15 acting on a particle at right angles to each other have a resultant of 68 lbs. ; find the fcs.

12. Shew by a diagram how forces of 37 and 50 lbs. can be applied to a particle so that their resultant will be 40 lbs.

13. The resultant of two fcs. is at right angles to one force, and is $\frac{1}{3}$ of the other ; compare the magnitude of the fcs.

14. The resultant of two fcs. makes an angle of 120° with one of the forces ; shew that it is less than the other force.

15. Find at what angle two equal forces must act so that their resultant may make an angle of 60° with one of the forces.

16. Two fcs. represented by 11 P and 60 P respectively act on a point at right angles to each other ; find the force which will balance them.

17. Shew that three fcs. acting at a point, but not in the same plane, cannot be in equillibrium.

18. Two diagonals of a parallelogram, AC, BD, intersect in O, fcs. acting on a particle are represented by AO, OD ; shew which side of the parallelogram represents their resultant.

19. A boat is moored in mid-stream by two ropes fastened to each bank, making an angle of 120° with each other. The force of the stream is 200 lbs. : find the tension in the ropes.

20. A boat is moored to two points on opposite banks, so that the line joining them is at right angles to the stream, and 85 ft. in length. The two ropes are at right

angles to each other, and one of them is 13 ft. long; find tension in each rope if the force of the stream is 170 lbs.

21. A smooth ring sustaining a weight of 30 $\sqrt{2}$ lbs. slides along a cord fastened at two points lying in same horizontal line; find tension of the cord when the two parts of the cord are at right angles to each other.

22. A weight of 75 lbs. is sustained by two cords whose lengths are 15 and 20 inches respectively, fastened to two points lying in the same horizontal plane; find tensions in the cords when they are at right angles to each other.

23. In Ex. 22, if length of cords were 21 and 220 inches, find tensions

(*b*) If tension of cord 15 in. was 80 lbs., find weight.
(*c*) " " 20 in. " 5 lbs., " "
(*d*) " " 15 in. " P lbs., find tension of other cord.

24. In Ex. 21 find tension in the cord when the two parts of the cord form an angle of 60°.

25. Three fcs. acting at a point are represented by three adjacent sides of a regular hexagon taken in order; find their resultant.

26. Three posts are placed in the ground so as to form an equilateral triangle, and an elastic ring is stretched round them, the tension of which is 12 lbs.; find pressure on each post.

27. A cord having equal weights attached to its extremities passes over pulleys placed at A and B in the same horizontal line 48 in. apart, and through a smooth ring C, from which a weight of 60 lbs. is attached; what must be the magnitude of the equal weights that C may rest exactly 10 inches below the line AB?

28. In Ex. 27, if $AB = 18$ in., C 12 inches below AB and weight $= 36$ lbs., find the tension of the cords.

(*b*) If $AB = 60$ inches, C 16 in. below AB, and equal weights, 85 lbs., find weight supported at C.

29. Find the horizontal and vertical pressures when a force of 120 lbs. acts in a direction of 60° with the vertical line.

30. A ball weighing 200 lbs. slides along a smooth rod inclined at an angle of 30° with the vertical line, what force acting along the rod will support the ball, and what is the pressure on the rod?

31. In Ex. 30, if the angle was (a) 45°, (b) 60°, find force and pressure.

32. What power acting along a smooth plane inclined at an angle of 30° to the horizon will sustain a weight of 24 lbs. on the plane?

33. A weight of 30 lbs. is suspended by two flexible strings, one of which is horizontal and the other inclined at an angle of 60° with the vertical; find tension in each string.

CHAPTER VI.

THE REACTION OF SMOOTH SURFACES AND HINGES.

19. The reaction of a smooth surface is always exerted at right angles to it.

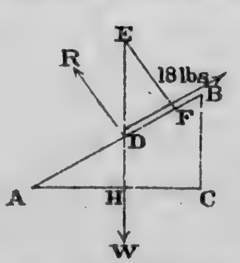
20. The reaction of a hinge passes through the point of intersection of the lines representing the direction of the other forces.

21. Whenever three forces which are not parallel are in equilibrium, their directions pass through the same point.

Lines representing directions of two of the forces must meet (since they are not parallel), and their resultant must pass through this point. The third force must be equal and opposite to this resultant in order to be in equilibrium, and therefore must pass through the point of intersection of the other forces.

EXERCISE VII.

1. What weight can be supported on a plane inclined at an angle of 30° to the horizon by a force of 18 lbs. acting along the plane?



Let AB represent the plane, BAC = 30°. Weight placed at D will act vertically downwards, the reaction R will be at right angles to the plane. Produce the direction of the weight W upwards, and from E any point in it let fall $\perp$ EF. Then triangle EDF has its sides parallel to the direction of the three forces, and $\therefore$ must represent them.

$\because$ the angle DAC = 30°, $\therefore$ ADH = 60° = EDF and DFE = 90, $\therefore$ DEF = 30°.

If ED = 2, DF = 1, and EF = $\sqrt{3}$.

$1 = 18$ lbs.

$2 = 36$ lbs. = weight.

$\sqrt{3} = 18\sqrt{3}$ lbs. = reaction of plane.

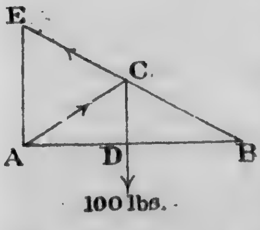
It is manifest that the triangle EDF in all such questions as above is always similar to triangle ABC, and

therefore if any two sides of triangle ABC is given, we know the ratio of the two corresponding sides of triangle EDF, and can find the ratio of the third side by Euc. I., 47.

2. What force acting along a plane rising 5 ft. in 13 ft. is necessary to support a weight of 52 lbs. resting on the plane?

$BC = 5$, $AB = 13$, $\therefore$ ratio of DF : DE is as 5 : 13.
If $13 = 52$ lbs.
$5 = 20$ lbs. = force required.

3. A beam AB weighing 100 lbs. is supported in a horizontal position by a hinge at A, and a rope fastened at B, making an angle of 30° with the beam; find the tension in the rope.



Let AB represent the beam, the weight 100 lbs. acting at its centre D. Produce the direction of weight upwards to meet direction of rope in C. Then the reaction of the hinge at A must pass through C. Through A draw AE parallel to CD, meeting BC produced in E. The sides of the triangle ACE represents the three forces.

$\because$ AB is horizontal, CD vertical, and AE parallel to CD. $\therefore$ EAB = 90°, and ABE = 30°, $\therefore$ AEB 60°; $\therefore$ if AE = 1, EB = 2, and $\because$ D is middle point if AB and DC is parallel to AE; $\therefore$ C is middle point of EB; $\therefore$ FC = 1.

AE = 1 = 100 lbs.
$\therefore$ EC = 1 = 100 lbs. = tension in the rope.

4. A sphere weighing 80 lbs. is supported on a plane inclined at an angle of 30° to the horizon by a force acting parallel to the plane; find the reaction of the plane.

5. In Ex. 4 if the reaction on the plane was 120 lbs. ; find the force and the weight.

6. A beam of uniform density weighing 240 lbs. is supported in a horizontal position by a hinge at A and a rope fastened at B, making an angle of 30° with the beam ; find tension in the rope and reaction of the hinge.

7. In Ex. 6, *(a)* If the beam made an angle of 30° with the vertical ; find tension in the rope.

(b) If it made an angle of 60°. *(c)* 120° ; find tension of rope.

8. A beam AB of uniform density weighing 300 lbs. is hinged at A, the other end B rests against a smooth vertical wall ; find the reaction of the hinge, and the wall, when the beam makes an angle of *(a)* 30°, *(b)* 45° *(c)* 60°, with the wall.

9. A rigid rod AB without weight, hinged at A, a cord fastened to end B is attached to a point C vertically over the hinge ; a weight of 30 lbs. is suspended from B ; find portion of the weight supported by the rod, and tension in the cord, when *(a)* cord BC = $\frac{1}{2}$ AC, *(b)* if BC = AC, *(c)* if BC = 2 AC. (The cord being horizontal in each case.)

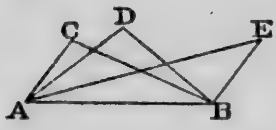
10. Find the least horizontal force necessary to draw a wheel whose weight is w, and radius r, over an obstacle the height of which is h, situated on the horizontal plane on which the wheel rests.

CHAPTER VII.

RESOLUTION OF FORCES.

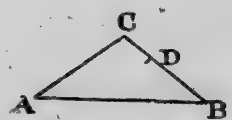
22. Resolution of fcs. is finding the component forces.

Two component forces can only have one resultant, but any force may be the resultant of any number of pairs of component forces. Thus AB may form the side of any number of triangles such as ACB, ADB, AEB, &c., and therefore it may represent the resultant of forces represented by AC and CB, or AD and DB, or AE and EB, ∴ we can only effect the resolution of a force, when we know the direction of the components.



23. To resolve a force into two other forces acting in given directions.

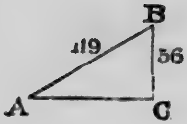
Let AB represent the direction of the given force. From A any point in AB draw AC in direction of one of the component, and from B any other point in AB draw BD parallel to the direction of the other component, and produce BD and AC to meet in C, then AC and CB shall represent components required.



24. When forces are resolved along two lines at right angles to each other, the sum of the forces acting in one direction must equal the forces acting in opposite directions, if there is equilibrium.

EXERCISE VIII.

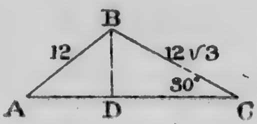
1. A force of 119 lbs. is resolved into two others acting at right angles, one of the components is 56 lbs.; required the other component.



Let AB represent 119 lbs. Draw AC horizontally and BC perpendicular. Let BC = 56 lbs.

$$\begin{aligned} AC^2 &= AB^2 - BC^2, \\ &= (119)^2 - (56)^2, \\ &= 11025, \\ \therefore AC &= 105 \text{ lbs.} = \text{other component.} \end{aligned}$$

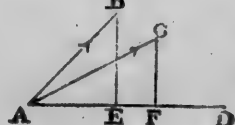
2. A force of 12 lbs. is resolved into two others acting at an angle of 30°; one of the components is $12\sqrt{3}$; find the other component.



Let AB represent 12 lbs., $\angle$ BCA = 30°, BC = $12\sqrt{3}$ lbs. Let fall perpendicular BD. Then if BC = 2, BD = 1, DC = $\sqrt{3}$; $\therefore$ if BC = $12\sqrt{3}$, BD = $6\sqrt{3}$ and CD = 18.

$$\begin{aligned} AD^2 &= AB^2 - BD^2. \\ &= 12^2 - (6\sqrt{3})^2 = 36; \ \therefore AD = 6, \text{ and } DC\ 18; \\ &\therefore AC = 24 = \text{other component.} \end{aligned}$$

3. Two equal forces act on a particle at an angle of 30°; find their resultant.



Let AB and AC represent the equal forces. Draw AD so that $\angle$ DAC = 30°; from B and C let fall perpendiculars BE and CF. Then $\because$ BAE = 60° if AB = P, AE = $\frac{P}{2}$ and BE = $\frac{P}{2}\sqrt{3}$.

Again $\because$ CAE = 30°, if AC = P, AF = $\frac{P}{2}\sqrt{3}$ and CF = $\frac{P}{2}$.

AE and AF act in same direction, and BE and CF act

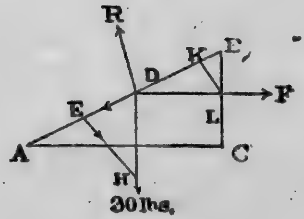
at right angles, $\therefore$ we have $\frac{P}{2} + \frac{P}{2}\sqrt{3}$ and $\frac{P}{2}\sqrt{3} + \frac{P}{2}$ acting at right angles to find their resultant.

$$R = \sqrt{\left(\frac{P}{2} + \frac{P}{2}\sqrt{3}\right)^2 + \left(\frac{P}{2}\sqrt{3} + \frac{P}{2}\right)^2}$$

$$= \sqrt{2\left(\frac{P}{2} + \frac{P}{2}\sqrt{3}\right)^2} = \sqrt{\frac{2P^2}{4}\left(1 + \sqrt{3}\right)^2}$$

$$= P\sqrt{2 + \sqrt{3}}.$$

4. What force acting horizontally will sustain a weight of 30 lbs. on a plane inclined to the horizon at an angle of 30° ?

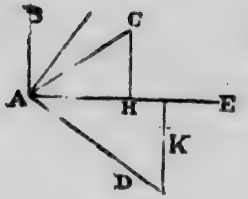


Let AB represent the plane. $\angle$ BAC = 30°. From H any point in direction of weight let fall perpendicular HE. Resolve 30 lbs. parallel and perpendicular to the plane, DE = 15 lbs. and EH = $15\sqrt{3}$. In same manner resolve F. DK = $\frac{F}{2}\sqrt{3}$ and KL = $\frac{F}{2}$.

$$\text{Then } \frac{F}{2}\sqrt{3} = 15 \quad \therefore \quad F = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

$$R = \frac{F}{2} + 15\sqrt{3} = 5\sqrt{3} + 15\sqrt{3} = 20\sqrt{3}.$$

5. Three forces of 10, $8\sqrt{3}$ and $22\sqrt{2}$ lbs. act at a point, and the angle included between the first and second is 30°, between the second and third 105°; find their resultant.



Let AB, AC and AD represent the forces. $\angle$ BAC $= 30°$, $\angle$ CAD $= 105°$. Draw AE at right angles to AB; then CAE $= 60°$ and EAD $= 45°$. Resolve forces parallel and at right angles to AE. If AC $= 8\sqrt{3}$, AH $= 4\sqrt{3}$ and HC $= 12$, if AD $= 22\sqrt{2}$, AK $= 22$ and KD $= 22$, AB $= 10$, HC $= 12$, $\therefore$ these forces equilibriate KD, which is 22, $\therefore$ R $=$ AH $+$ AK $= 4\sqrt{3} + 22 =$ 28.928 lbs.

6. If a force of 80 lbs. be resolved into two equal forces acting at right angles, what is the magnitude of either component?

7. If a force of 175 lbs. be resolved into two fcs. acting at right angles, one of which is 49 lbs., find the other component.

8. If a force of P lbs. be resolved into two fcs. acting at right angles, find the components when one is three times the other.

9. Resolve a force of 50 lbs. into two equal forces acting at an angle of 30°.

10. Resolve a force of 150 lbs. into two equal fcs. acting at an angle of 60°.

11. Resolve a force of 200 lbs. into two equal fcs. acting at an angle of 135°.

12. Resolve a force of 60 lbs. into two equal fcs. acting at an angle of 45°.

13. Resolve a force of 80 lbs. into two equal fcs. acting at an angle of 120°.

14. Resolve a force of 120 lbs. into two equal fcs. acting at an angle of 150°.

15. The resultant of two fcs. acting at an angle of 120° is 126 lbs., one of the components is 54 lbs.; find the other component.

16. The resultant of two fcs. acting at right angles is 50 lbs., and makes an angle of 60° with one of the components; find the components.

17. A force of 18 lbs. is resolved into two others acting at an angle of 30°; one of the components is $18\sqrt{3}$; find the other component.

18. If a cord 27 inches in length be fastened to two points in the same horizontal line, 18 in. apart, a weight of 30 lbs. is supported by means of a smooth ring sliding on the cord, find the tension in the cord.

19. Two equal fcs. act on a particle at an angle of 60°; find the resultant.

20. Two equal fcs. act on a particle at an angle of 135°; find the resultant.

21. Three fcs. of 12, 10 and 8 lbs. act on a particle and include angles of 75° and 60°; find their resultant.

22. Three equal fcs. act on a particle, and the angle included between the first and second is 45°, between the second and third 75°; find the resultant.

23. A man pulls at a weight by means of a rope along a road with a force of 120 lbs., the rope making an angle of 60° with the road; find with what force a boy would require to pull in the opposite direction with a rope making an angle of 30° with the road, so as to neutralize the force exerted by the man.

24. Two men pull a heavy weight by ropes inclined to the horizon at angles of 45° and 60°, with a force of 90 and 100 lbs. respectively. The angle between the two vertical planes of the cords is 135°; find the horizontal force produced.

25. Find the resultant of two fcs. of 20 and 24 lbs. acting on a point at an angle of 15°.

26. Find the horizontal and vertical pressures when a force of 120 lbs. acts in a direction making an angle of *(a)* 30°, *(b)* 45°, *(c)* 60° with the vertical line.

27. Forces of 6, 8, 10 and 12 lbs. respectively act along lines drawn from the centre of a square to the angular points taken in order; find their resultant.

28. Six points are taken on the circumference of a circle at equal distances, and from one of the points straight lines are drawn to the rest; if these straight lines represent forces acting at the point, shew that the resultant is three times the diameter of the circle.

29. Forces of 6, 8 and 10 lbs. respectively act along lines drawn from the centre of a circle to the angular points of the inscribed equilateral triangle; find their resultant.

30. Three fcs. of 60, 70 and 80 lbs. respectively act on a particle, making angles of 120° with each other successively; find the magnitude of the resultant.

31. Find the resultant of three fcs., the least of which is 12 lbs., which are represented by and act along OA, OB, OC, two sides and the diagonal of a rectangle whose area is 90 sq. inches, the sides being in ratio of 2 : 5.

32. What force acting parallel to a smooth plane inclined to the horizon at an angle of 30°, will sustain a weight of 10 lbs. on the plane?

33. If in Ex. 32 the weight of 10 lbs. is placed at the middle point of the plane, and is kept at rest by a string passing through a groove in the plane and attached to the opposite extremity of the base; find the tension in the string.

34. A cord capable of sustaining a tension of 8 lbs. is fastened at one end to a point in an inclined plane, a weight of 16 lbs. is attached to the other end, the inclination of the plane being gradually increased; find when the cord will break.

35. What force acting horizontally will sustain a weight of 60 lbs. on a plane inclined to the horizon at an angle of *(a)* 30°, *(b)* 45°, *(c)* 60°?

36. The length of an inclined plane is 10 ft. and the height is 5 ft; find into what two parts a weight of 80 lbs. must be divided so that one part hanging over the top of the plane may balance the other part resting on the plane.

37. A weight of 12 lbs. is supported on a plane inclined at an angle of 60° with the horizon by two forces, one acting parallel to the plane, and the other horizontally; find the ratio of the forces when the reaction is 12 lbs.

38. A weight of 30 lbs. is supported on a plane whose inclination is 30°, by three equal forces, one acting horizontally, one parallel to the plane, and the other at an angle of 30° with the plane; find the equal forces and the reaction of the plane.

39. Two planes of equal altitude are inclined at angles of 30° and 45° to the horizon; what weight resting on the former will balance 60 lbs. on the latter, the weights being connected by means of a string passing over their common vertex?

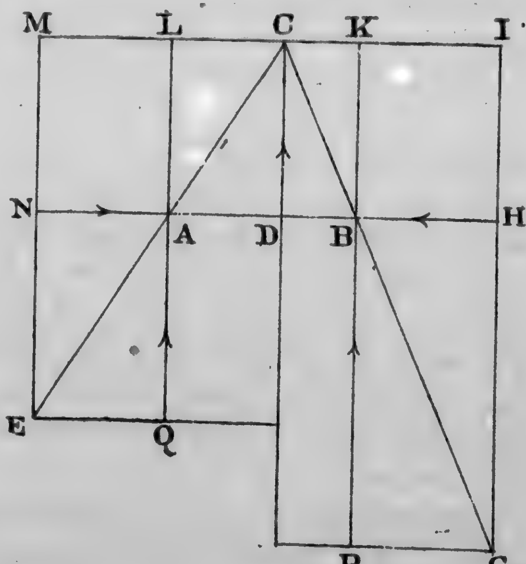
CHAPTER VIII.

PARALLEL FORCES.

25. **Parallel forces** are those acting at different points of a body and in directions parallel to one another.

26. The **resultant** of two parallel forces is the sum of their separate effects if they act in the same direction; and is equal to their differences if they act in opposite directions; *i. e.*, it is equal to their algebraical sum.

27. The **position** of the resultant may be deduced from the parallelogram of fcs. as follows:—



Let P and Q be any parallel forces represented by QA and PB. Draw any line AB at right angles to

their directions. At A and B let two equal and opposite forces be represented by NA and HB. The resultant of NA and QA is EA, and of HB and PB is GB, produce EA and GB to meet in C. Resolve these into their original components acting at C, we shall have two equal and opposite forces acting in line IM corresponding to NA and HB (which will be in equilibrium, and therefore will have no effect upon the other forces), and two forces equal to P and Q acting along DC,

Therefore their resultant is equal to $P + Q$ acting along DC.

The parallelograms QD and NL are equal (Euc. I., 43); so, also, $PD = BI$. But $NL = BI$ (Euc. I., 36), $\therefore QD = PD$. But QD is a rectangle, and therefore equals $QA \cdot AD$; so, also, $PD = PB \cdot BD$.

$$\therefore QA \cdot AD = PB \cdot BD.$$
$$i.e., Q \cdot AD = P \cdot BD.$$

That is the point D divides AB into segments which are inversely as the forces Q and P respectively.

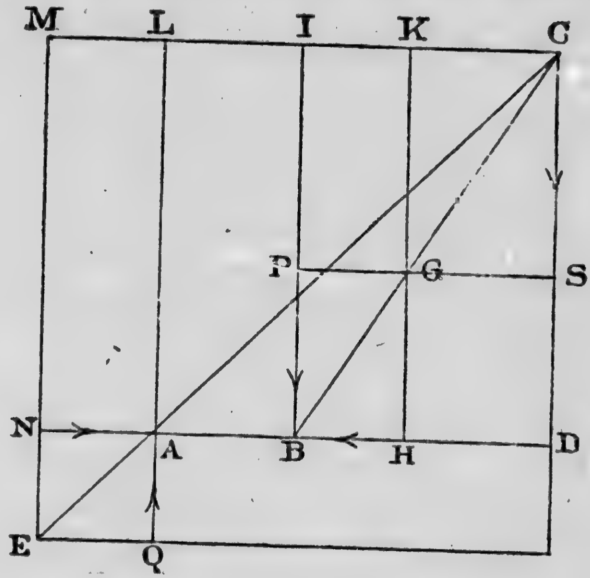
If the fcs. act in contrary directions, the resultant will pass outside the greater force.

Using the same construction as the preceding, we have

$$QD = MA = BK = PD.$$
$$\therefore QD = PD.$$
$$Q . AD = P . BD.$$
$$i.e., P : Q :: AD : BD.$$

28. If the force P was equal to the force Q, then

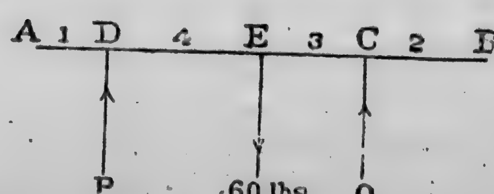
EA would be parallel to BG, and the point C would



be infinitely distant; *i. e.*, there is no single force that would produce the same effect as that produced by P and Q. Such a pair of forces is called a couple. The effect of a couple is to produce rotation.

EXERCISE IX.

1. A uniform beam AB, 10 ft. long, weighing 60 lbs., is supported on two props 1 ft. and 2 ft. from the ends A and B, respectively; find the pressure on each prop.



Let AB represent the beam; props placed at D and

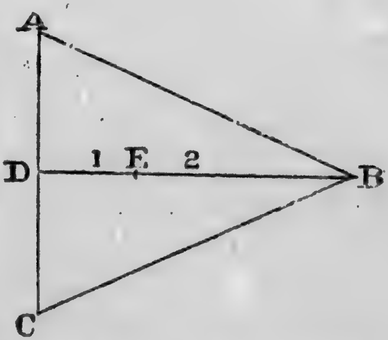
C, 1 ft. and 2 ft. from A and B. Let P represent weight supported at D, and Q that supported at C.

$$\text{Then } P \cdot DE = Q \cdot CE.$$
$$P \cdot 4 = Q \cdot 3.$$
$$\therefore P = \tfrac{3}{4}Q$$

and $P + Q = 60$, $\frac{3}{4}Q + Q = 60$, $\therefore Q = \frac{4}{7}$ of $\frac{60}{1} = 34\frac{2}{7}$ lbs., and $P = 25\frac{5}{7}$ lbs.

2. A weight of 90 lbs. rests on a horizontal table at a point in the line joining an angular point with the middle of the opposite side, and at a distance from the angular point equal to twice its distance from the side; find the pressure on each leg, supposing the legs to be at the angular points.

Let ABC represent the table, and BE = 2 DE and AD = DC. 90 lbs. at E is the same as 60 at D and 30 at B; $\because$ the weights are inversely as the distance from the resultant, and are $\therefore$ as 2 : 1.



Again, 60 lbs. at D is equivalent to 30 at A and 30 at C, $\because$ D is middle point of AC; $\therefore$ 90 lbs. at E is equivalent to 30 lbs. at A, B and C; $\therefore$ pressure on each leg is 30 lbs.

3. Find the position of the resultant of two parallel fcs. acting in the same direction, whose magnitudes are 25 lbs. and 49 lbs. respectively, acting at points 37 in. apart.

4. The smaller of two parallel fcs. acting in the same direction is 20 lbs., the resultant is 50 lbs. acting at a

distance of 12 inches from the larger force ; find the distance between the forces.

5. Two men of the same height carry on their shoulders a pole 15 ft. long, to which a weight of 180 lbs. is attached, at a distance of 5 ft. from one of the men ; what portion of the weight does each man support ?

6. Two weights of 8 lbs. and 10 lbs. hang at the ends of a rod 6 ft. long, and a third weight of 6 lbs. is placed 20 inches from the 8 lbs. ; find the position of the resultant.

7. The larger of two parallel fcs. acting in contrary directions is 7 lbs., the resultant is 4 lbs. acting at a distance of 6 inches from the larger force ; find the distance between the forces.

8. A weight of 8 lbs. hung from one extremity of a straight lever balances a weight of 20 lbs. hung from the other ; find the ratio of the arms.

9. Two weights, which together weigh 12 lbs., are suspended at the ends of a straight lever and balance ; if the fulcrum is 3 times as far from one end as from the other, find each weight.

10. A uniform beam 10 ft. long, weighing 20 lbs., is supported by two props at the ends of the beam ; find where a weight of 40 lbs. must be placed so that the pressure on the two props may be 15 lbs. and 45 lbs. respectively.

11. 10 lbs. is suspended from each angle of a triangle ; find the point at which it must be suspended to rest horizontally.

12. Two forces of 20 and 25 lbs. acting in contrary directions are 10 inches apart ; find position and magnitude of their resultant.

13. Parallel fcs. of 8 and 12 lbs. act in the same direction, at two points A and B in a body ; find at what point in AB a single force must act to maintain equilibrium.

14. A man supports two weights fastened to the ends of a stick 5 ft. long placed across his shoulder ; find the point of support, if one weight be $\frac{7}{8}$ of the other.

15. A board 18 ins. square is suspended in a horizontal position by strings attached to its four corners, A, B, C, D, and a weight of 20 lbs. is laid upon it at a point 6 ins. distant from the side AB and 8 ins. from AD ; find the tension in each string, disregarding the weight of the board.

16. A rectangular slab 9 ins. by 16 ins., weighing 40 lbs., is suspended in a horizontal position by strings fastened to its four corners, A, B, C, D, and a weight of 60 lbs. is placed upon it at a point 3 ins. from the side AD and 4 ins. from DC ; find the tension in each string.

17. If the tensions in the string in Ex. 16 were 8, 12, 16 and 20 lbs. respectively ; find the position of the resultant.

CHAPTER IX.

FORCES PRODUCING ROTATION—MOMENTS.

29. If a point in a body be fixed, but the body free to rotate about that point, a force applied at any other point, and in a direction not passing through the fixed point, will produce rotation.

30. The rotary effect depends on the magnitude of the force and on the distance of its direction from the fixed point.

31. The tendency of a force to produce rotation about a point is called its **moment** about that point.

32. The moment of a force about a point is measured by the product of the force into the perpendicular distance of its direction from the point.

Hence, if the direction of the force passes through the given point, its moment about the point is nothing.

33. The moment of a force may be represented geometrically by twice the area of the triangle having the straight line representing the force for its base, and the point about which the moment is taken for its vertex, since the base multiplied by the perpendicular gives twice the area of the triangle.

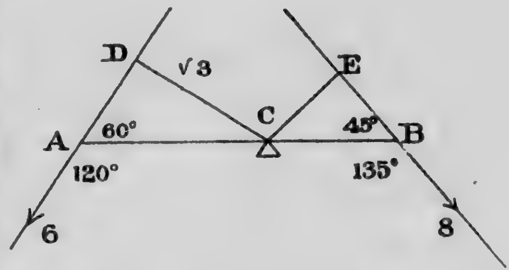
34. **Equilibrium of Moments.**—If any number of forces act upon a body, they will produce equilibrium when their moments about any point are equal and opposite. For if their moments about any point were not equal and opposite, they would produce rotation of the body about that point.

35. If we consider the moments in one direction positive, and those in the opposite direction negative; then in order that these may be equilibrium, the algebraic sum of the moments of the forces about *any* point must be zero.

The equilibrium of moments may be proved geometrically, for which we would refer the student to a more advanced work.

EXERCISE X.

1. Two fcs. of 6 lbs. and 8 lbs. act at the extremities of of a straight lever 2 ft. long, and inclined to it at angles of 120° and 135° respectively; find the position of the fulcrum when there is equilibrium.



Let AB represent the lever, whose fulcrum is C. Produce the direction of the 6 and 8 lbs. backwards, and let fall the perpendiculars CD and CE. Then the moment of 6 lbs., about C $=6$CD, and moment of 8 lbs. about C $=8$.CE. Since there is equilibrium 6 CD $=8$ CE, *i.e.*, CD $=\frac{4}{3}$ CE.

If CD $=\sqrt{3}$, then AC $=2$. $\therefore$ if CD $=\frac{4}{3}$ CE, AC

$$=\frac{8}{3\sqrt{3}}\text{ CE.}$$

Again, if CE $=1$, then BC $=\sqrt{2}$, $\therefore$ BC $=$ CE $\sqrt{2}$. $\therefore$ the fulcrum divides AB in ratio of $\frac{8}{3\sqrt{3}}:\sqrt{2}$ or

$$8:3\sqrt{6}.$$

Or let AC $=x$, BC $=y$, $\therefore x+y=2$ ft.

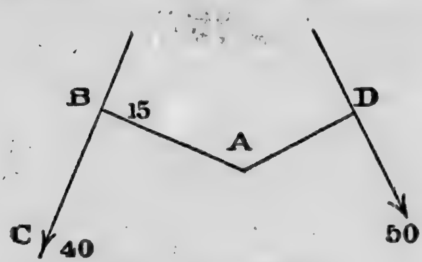
$$\text{CD}=\frac{x\sqrt{3}}{2},\quad \text{CE}=\frac{y}{\sqrt{2}}\text{ and } 6.\text{ CD}=8.\text{ CE}$$

$$i.\,e.,\ \frac{6.\sqrt{3}.\,x}{2}=\frac{8\,y}{\sqrt{2}},\ \therefore\ \frac{x}{y}=\frac{16}{6\sqrt{6}}=\frac{8}{3\sqrt{6}}$$

$$\frac{2-y}{y}=\frac{8}{3\sqrt{6}},\ 2-y=\frac{8\,y}{3\sqrt{6}},\ y=\frac{2}{1+\frac{8}{3\sqrt{6}}}$$

$$=\frac{6\sqrt{6}}{8+3\sqrt{6}}=\text{BC.}$$

2. A force of 40 lbs. acts at a distance of 15 inches from a fixed point, at what distance must a force of 50 lbs. act, in order that there may be equilibrium?

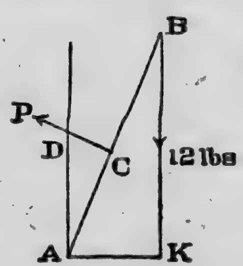


Let A be the fixed point and BC = direction of 40 lbs., and DE direction of 50 lbs.; let fall perpendiculars AB and AD. AB is given 15 ins.

Then 40 . AB = 50 AB.

$$40 \,.\, 15 = 50 \,.\, AD \;\therefore\; AD = \frac{40 \,.\, 15}{50} = 12 \text{ ins.}$$

3. A straight rod movable about one end makes an angle of 30° with the vertical. A weight of 12 lbs. hangs at the other end; what force acting perpendicularly to the rod at its middle point will produce equilibrium?



Let AB = rod, and angle BAD = 30°, ∴ BAK = 60°.

If AK = 1, AB = 2, and AC = 1.

Take moments about A.

$$12 \,.\, AK = P \,.\, AC.$$
$$\therefore\; P = 12 \text{ lbs.}$$

NOTE.—There is a reaction at A, and by taking moments about that point, its moment vanishes, no matter what the force. It is generally advisable to take moments about some point through which some of the unknown forces pass, in order to diminish the number of equations.

4. Forces of 3 and 9 lbs. act in parallel directions at two points 20 ins. apart; find the magnitude and point of application of the force that will equilibriate them.

5. A rod 3 feet long and weighing 10 lbs. is supported

by a fulcrum 1 foot from one end ; find what weight must be attached to that end so as to be in equilibrium.

6. A rod 3 feet long and weighing 8 lbs. has a weight of 4 lbs. placed at one end ; find at what point the rod must be supported so as to be in equilibrium.

7. If the forces at the ends of the arms of a horizontal lever be 10 and 11 lbs., and the arms 10 ins. and 11 ins. respectively, find at what point a force of 3 lbs. must be applied at right angles to the lever to keep it at rest.

8. The length of a horizontal lever is 6 ft. and the balancing weights at its ends are 4 and 6 lbs. ; if each be moved 15 ins. from the end of the lever, find how far the fulcrum must be moved for equilibrium.

9. ABC is a triangle without weight having a right angle at C, and CA : CB :: 5 : 3 ; two forces P and Q acting at A and B in directions at right angles to CA and CB, keep it at rest ; find the ratio of P to Q.

10. The lower end B of a rigid rod without weight 15 ft. long is hinged to an upright post, and its other end A is fastened by a string 12 ft. long to a point C vertically above B, so that ACB is a right angle. If a weight of 8 cwt. be suspended from A, find the tension of the string.

11. ABC is a bent lever without weight of which B is the fulcrum, and the angle ABC is 120° ; a weight P acting at A is supported by a weight of 10 lbs. at C, when AB is horizontal ; find the weight at C necessary to support P, when BC is horizontal ; the arms being of equal length.

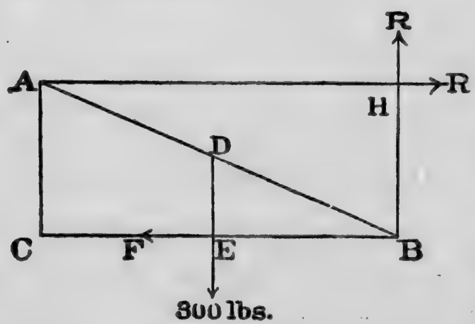
12. In Ex. 11, if the angle was *(a)* 150°, *(b)* 135°, find the weight.

13. A uniform beam 6 ft. long, the weight of which is

12 lbs., balances across a prop, with 6 lbs. hanging from one end and 15 lbs. from the other; find the point at which the beam is supported. If the weight at the two ends change places. what weight must be added to the lighter to preserve equilibrium?

EXERCISE XI.

1. A uniform beam AB 37 ft. long, weighing 300 lbs., rests with one end against a smooth wall, and the other end on a smooth floor, this end being tied by a cord 35 ft. long to a peg at the bottom of the wall; find the tension of the cord.



Let AB represent the beam; its weight acting at its middle point D; BC = cord = 35 ft.

Draw BH at right angles to BC to denote reaction of the floor, and AH at right angles to AC to denote reaction of the wall.

$$AC = \sqrt{AB^2 - BC^2} = \sqrt{37^2 - 35^2} = 12 = BH.$$

$\because$ D is middle point of AB and DE is // to AC, $\therefore$ E is middle point of BC.

$\therefore$ BE $= \frac{35}{2}$. Let F denote tension of cord BC.

The forces acting on the beam are the weight, the two reactions and the tension of the cord. Take moments about H where the two reactions cut each other.

$$300 \cdot BE = 12 F.$$
$$300 \cdot \tfrac{35}{2} = 12 F, \therefore F = 437\tfrac{1}{2} \text{ lbs.}$$

Since the forces are at right angles to each other, therefore those acting in one direction must equal those acting in the opposite direction.

$$\therefore\ R = 300 \text{ lbs. and } R' = 437\tfrac{1}{2} \text{ lbs.}$$

2. A uniform beam 37 ft. long, weighing 400 lbs., rests with one end against a smooth wall, and the other end on a smooth floor, this end being fastened by a cord 12 ft. long to a peg at the bottom of the wall; find the tension of the cord.

3. In Ex. 2, if the weight of the beam acted at a point $\frac{1}{3}$ of its length from the lower end; find the reactions of the wall and floor.

4. A beam AB. weighing 120 lbs., acting at its middle point, is made to rest against a smooth vertical wall and on a smooth floor, by a force applied horizontally to the foot; find the force if the inclination of the beam is *(a)* 30°, *(b)* 45°, *(c)* 60°.

5. In Ex. 4, if a weight of 90 lbs. was suspended on the beam at a point *(a)* $\frac{1}{3}$ of its length from the foot, *(b)* $\frac{3}{4}$ of its length from the foot; find the horizontal force, the inclination of the beam being *(a)* 30°, *(b)* 45°, *(c)* 60°.

6. In Ex. 4, if the weight of the beam acted at a point $\frac{2}{3}$ of its length from the foot; find the force, the inclination of the beam being *(a)* 30°, *(b)* 45°, *(c)* 60°.

7. Taking the inclination of the beam as in Ex. 4, find where a weight of 120 lbs. must be suspended so that the horizontal force may be the same as that in Ex. 5; the beam being uniform and weighing 120 lbs.

8. A uniform beam, weighing 60 lbs., rests with one end against a peg in a smooth horizontal plane, and the

other end on a wall. The point of contact with the wall divides the beam into parts as 3 : 8 ; find the pressure on the peg, and the reaction of the wall, the inclination of the beam being *(a)* 30°, *(b)* 45°, *(c)* 60°.

CHAPTER IX.

CENTRE OF GRAVITY.

36. The centre of gravity may be defined as that point at which the whole weight of a body may be supposed to act.

Every body is composed of particles, each of which is attracted towards the centre of the earth, and these forces may be taken as parallel. The resultant of this system of parallel forces is the weight of the body, and the point at which this resultant acts is called the centre of gravity of the body. If the body be supported at this point, it will rest in any position.

37. Therefore every body has a centre of gravity, and can have but one centre of gravity.

38. If a body suspended from any point be at rest, the centre of gravity is in the vertical line drawn through this point. For the body is acted on by two forces, viz., the weight of the body, which acts vertically through the centre of gravity, and the reaction of the fixed point. But two forces cannot be in

equilibrium, except they are equal and in opposite directions. Therefore the vertical line through the centre of gravity must pass through the fixed point. The centre of gravity may be either above or below the fixed point.

39. **Centre of Gravity determined Experimentally.**—Suspend a body from any point and draw a vertical line through this point; suspend it from another point and draw another vertical line. Each of these lines must pass through the centre of gravity, therefore the point of their intersection is the centre of gravity of the body.

40. **Centre of Gravity of Homogeneous and Symmetrical Bodies.**—The geometrical centre of any perfectly symmetrical figure must be its centre of gravity, because there is no reason why the centre of gravity should be on one side of that point more than upon another. Hence,

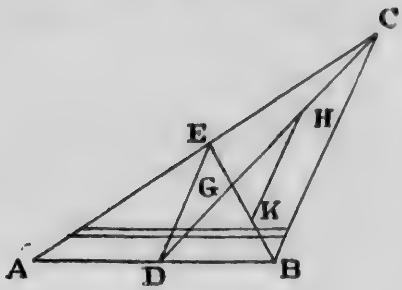
The centre of gravity of a straight line is its middle point.

The centre of gravity of a circle, or its circumference, or of a sphere, or of its surface, is its centre.

The centre of gravity of a parallelogram, or its perimeter, or of a parallelopiped, or its surface, is the point in which the diagonals intersect.

The centre of gravity of a cylinder, or of its surface, is the middle point of its axis.

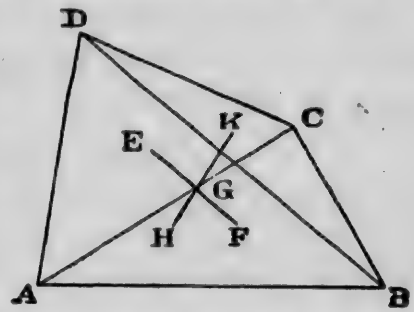
41. **To find the Centre of Gravity of a Triangle.**—Let ABC be a triangle. Draw the median line CD. If we suppose the triangle to be made up of an infinite number of lines parallel to AB, these lines will be bisected by CD. The centre of gravity of each of these lines being in CD, it follows that the centre of gravity of the triangle is in the line CD. In the same manner the centre of gravity must lie in the median line BE. Therefore the point G, where the two medians intersect, is the centre of gravity of the triangle.



Bisect GC in H and BG in K, join HK, since the line joining the middle points of two sides of a triangle is parallel and equal to one-half the base. $\therefore$ DE and HK are each parallel and equal to one-half BC. $\therefore$ they are parallel and equal to one another. $\therefore$ EG = GK and DG = GH (Euc. I., 26). But GH = HC and GK = KB. $\therefore$ DG = $\frac{1}{3}$ CD and EG = $\frac{1}{3}$ BE, *i.e.* the centre of gravity of the surface of a triangle is situated on a median line, at a distance of $\frac{1}{3}$ its length from the base.

42. **To find the Centre of Gravity of any Quadrilateral Figure.**—Let ABCD be any

quadrilateral figure, join AC, and find centre of gravity of the triangles ADC and ACB. let E and F be centre of gravity respectively. Then centre of gravity of whole figure must lie in EF. Similarly by joining DB, and finding H and K, the C. G. of the triangles ADB and DCB, the C. G. of whole figure must lie in the line HK. ∴ it must be at the point G where EF and HK intersect.



By proceeding in a similar way the centre of gravity of any rectilinear figure may be determined.

43. A body placed on a horizontal plane will stand or fall according as the vertical line through its centre of gravity falls within or without the base. The weight of a body acting vertically through the C. G. will be met by the resistance of the plane, if this line falls within the base, and therefore will have no tendency to move the body. But if the vertical line falls without the base, the weight of the body is not met by the resistance of the plane, and therefore the body will fall over.

44. **Different Kinds of Equilibrium.**—A body is said to be in **stable** equilibrium when, after a slight disturbance, it returns to its former position.

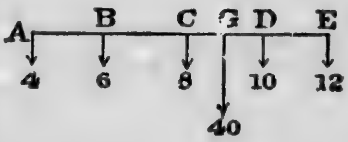
If it does not return to its former position, but

tends to remove further from its original position, it is said to be in **unstable** equilibrium.

If it remains at rest in the new position into which it was brought by the disturbance, it is said to be in **neutral** equilibrium.

EXERCISE XII.

1. Weights of 4, 6, 8, 10 and 12 lbs. act at points A, B, C, D and E in the same straight line 6 inches apart; find their centre of gravity.

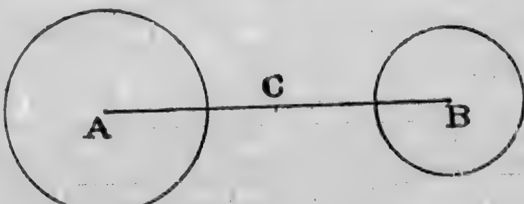


Let AE represent the straight line, forces acting at A, B, C, D and E. Let G denote their centre of gravity, at which the sum of their weights, or 40 lbs., may be supposed to act.

Then the moment of the whole force must be equal to the sum of its parts about any point. Take moments about E, and let x = distance of G from E; then

$$40\,x = 4 \times 24 + 6 \times 18 + 8 \times 12 + 10 \times 6.$$
$$= 96 + 108 + 96 + 60.$$
$$= 360.$$
$$\therefore\ x = 9 \text{ inches from point E.}$$

2. Two balls, weighing 8 and 10 lbs. respectively, are connected by a uniform rod weighing 3 lbs.; find C. G., the centre of the balls being 2 ft. apart.



Let A and B represent the centres of the alls, C the centre of the rod. Let x = distance of C. G. rom A, the centre of the ball weighing 10 lbs.:

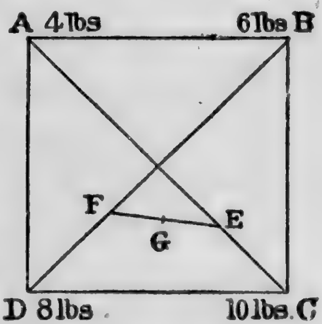
$$\text{then } 21\,x = 3 \times 1 + 8 \times 2$$
$$= 3 + 16$$
$$\therefore x = \tfrac{19}{21} \text{ ft. from A.}$$

3. Two uniform rods are joined so as to form the letter L, their lengths being as 2 : 7; find their C. G., disregarding their thickness.

Bisect AB in D and AC in E; join DE. The weights of the rods may be supposed to act at D and E, and to be as 7 : 2. Take DG $\frac{2}{9}$ of DE, and G will be C. G. required.

4. Four weights of 4, 6, 10 and 8 lbs. are suspended from the angular points of a square; find their C. G.

Let ABCD represent square; 4 lbs. at A and 10 lbs. at C is equivalent to 14 lbs. at E, a point $\frac{2}{7}$ of AC from C; 8 lbs. at D and 6 lbs. at B is equivalent to 14 lbs. at F, a point in DB, $\frac{3}{7}$ of DB from D; join FE. Then C. G. of 14 lbs. at F and 14 lbs. at E. is at the middle point G of the line EF.



Or let x = distance of C. G. from side AB; take moments about the side AB, and let a = side of square

$$28\,x = 8\,a + 10\,a$$

$\therefore x = \frac{9}{14}\,a$, i.e., C. G. is in a line parallel to AB, and at a distance from it equal to $\frac{9}{14}$ of the side the square.

Take moments about the side BC, and let y = distance of C. G. from BC.

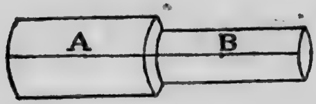
$$28\,y = 4\,a + 8\,a$$

$\therefore y = \frac{3}{7}\,a$, i.e., C. G. is in a line parallel to BC,

and at a distance from it equal to $\frac{3}{7}$ of the side of the square ; $\therefore$ C. G. must be where these two lines intersect.

5. Find the C. G. of two right cylinders having a common axis ; the radius of the one being 3 inches, and the altitude 8 inches ; the radius of the other being 2 inches, and its altitude 10 inches.

The volume of a cylinder is $= \pi\ r^2 h$

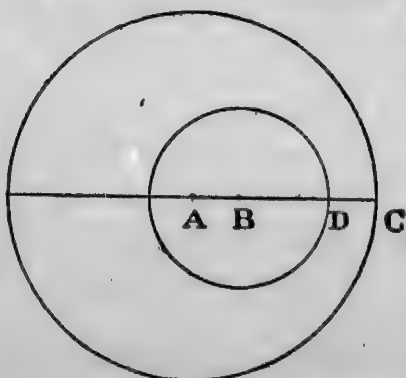


$\therefore$ the volume of one $= \pi\ 3^2.\ 8 = 72\ \pi$

$\therefore$ the volume of other $= \pi\ 2^2.\ 10 = 40\ \pi$

Their weights are proportional to their volumes, i.e., as $72\ \pi : 40\ \pi$, or as $9 : 5$. The C. G. of large cylinder is at A, its middle point, and of smaller at B, and AB $= \dfrac{8\ +\ 10}{2} = 9$ ins. Let $x =$ distance of C. G. from A, then taking moments about A, $14\ x = 45\ \therefore\ x = 3\frac{3}{14}$ ins., i.e., the C. G. of the two cylinders is at a point in their common axis $3\frac{3}{14}$ ins. from the centre of the larger.

6. From a uniform circular disc, another disc, having for its diameter the radius of the first circle, is cut away ; find the C. G. of the remainder, the centre of the smaller disc being $\frac{1}{2}$ its radius from the centre of the larger.



Let A be the centre of the larger disc, and B the centre of the smaller one, which is cut away. Let AC $= a$, then

$$BD = \frac{a}{2}, \text{ and } AB = \frac{a}{4}.$$

The area of the large disc $= \pi r^2 = \pi a^2$.

" " smaller disc $= \pi \dfrac{a^2}{4}$.

$\therefore$ " " remainder $= \pi \left(a^2 - \dfrac{a^2}{4}\right) = \pi \dfrac{3a^2}{4}$.

Take moments about A, and let C. G. of rem. be at a distance of x from A. Since moment of the whole must equal the sum of the moments of the parts about any point, we have

$$\pi a^2 \times 0 = \pi \frac{a^2}{4} \times \frac{a}{4} + \pi \frac{3a^2}{4} x,$$

$$0 = \frac{a^3}{16} + \frac{3a^2}{4} x,$$

$$\tfrac{3}{4}x = -\frac{a}{16}, \quad \therefore x = -\frac{a}{12},$$

i.e., the C. G. of the rem. is on the other side of A at a distance of $\frac{1}{12}$ of the radius of the larger disc.

7. Three weights of 8, 12 and 16 lbs. respectively, are placed at intervals of 9 ins. along a weightless rod ; find their C. G.

8. Find the C. G. of 15 and 20 lbs, the line joining their centres being 3 ft.

9. Find the C. G. of two weights of 8 and 12 lbs. suspended from the ends of a uniform horizontal rod 6 ft. long and weighing 10 lbs.

10. Two balls weighing 20 and 30 lbs. are connected by a uniform rod weighing 5 lbs. ; find the C. G., the centres of the balls being 2 ft. 6 ins. apart.

11. BC is the base of an isosceles triangle ABC whose height is 16 ins. ; weights of 20, 20 and 30 lbs. are suspended from B, C and A respectively ; find their C. G.

12. In Ex. 11 if the isos. triangle weighed 10 lbs. ; find the C. G.

13. A uniform rod 5 ft. long, weighing 12 lbs., balances about a point, 10 ins. from one end to which a weight W is attached ; find W.

14. A rod 3 ft. long, weighing 8 lbs., has weights of 3, 5, 7, 9 and 12 lbs. placed along it at intervals of 9 ins. ; find the C. G.

15. Three weights of 8, 10 and 15 lbs. are at equal intervals of 12 ins. on a uniform lever 2 ft. long, weighing 12 lbs. ; find the position of the fulcrum in order that there may be equilibrium.

16. A uniform bar weighing 10 lbs. balances about a point 3 ins. from the middle, having weights of 12 and 15 lbs. respectively, suspended from the ends ; find the length of the bar.

17. Three uniform rods are joined so as to form the letter F ; find their C. G., neglecting their thickness, their lengths being as 1 : 2 : 5.

18. Three uniform rods, each 4 ft. long, are joined so as to form an equilateral triangle ; find their C. G.

19. Three uniform rods, each 6 ft. long, form three adjacent sides of a square ; find their C. G.

20. Find the C. G. of the quadrilateral figure formed by two isosceles triangles upon opposites sides of a common base, their altitudes being h_1 and h_2.

21. Weights of 7, 8, 9, 4, 11 and 6 lbs. are placed at the corners of a regular hexagon taken in order ; find their C. G.

22. A circular table weighing 60 lbs. rests on four

legs, in its circumference forming a square; find the least pressure that must be applied at any point to overturn it.

23. A circnlar table weighing 60 lbs. rests on three legs, in its circumference forming an equilateral triangle; find the least weight that placed at any point on the table will overturn it.

24. Three uniform rods are placed so as to form a right-angled isosceles triangle, the longest being $8\sqrt{2}$ ft.; find their C. G.

25. A heavy tapering rod, weighing 160 lbs., balances about a point 4 ft. from the heavy end, when a weight of 30 lbs. is attached to the other end; find the point about which it will balance if the 30 lbs. is removed.

26. Four weights of 5, 7, 9 and 11 lbs. are placed at the corners of a square plate weighing 12 lbs., whose side is 10 inches; find the distance of the C. G. from the centre of the plate.

27. Weights of 4, 6, 8, 7, 3, 2, 13 and 1 lbs. are placed at the corners and middle points of the sides of a square taken in order; find their C. G., the side of the square being 16 inches.

28. A square plate of uniform thickness, whose side is 2 ft., is divided into two portions by a diagonal, the one part weighing 20 lbs. and the other 30 lbs.; find the distance of the C. G. from the geometrical centre.

29. A uniform square plate whose side is 10 inches and weight 12 lbs., has a weight of 18 lbs. attached to one corner; where must it be suspended by a cord so as to rest horizontally?

30. A tower is built in the form of an oblique cylinder, the diameter of whose base is 24 ft., and for every foot

it rises it inclines 2 ins. from the vertical; find the greatest height it can have.

31. A right cylinder whose diameter is 8 inches can just rest upon an inclined plane; find its height when the inclination of the plane is (*a*) 30°, (*b*) 45°, (*c*) 60°.

32. An isosceles triangle whose base is 4 ft. rests upon an inclined plane; what is the greatest height the triangle can have without toppling over; when the inclination of the plane is (*a*) 30°, (*b*) 45°, (*c*) 60°?

33. Find the centre of gravity of two right cylinders having a common axis, the radius of the one being 4 inches, and the altitude 9 inches; the radius of the other being 3 inches, and its altitude 12 inches.

34. A cylinder, whose diameter is 8 ft. and height 20 ft., rests on another cylinder, whose diameter is 12 ft. and height 10 ft.; find their centre of gravity when their axes coincide.

35. Into a hollow rectangular vessel, 1 foot high, a uniform solid, 6 inches long, is just fitted; find their common centre of gravity if the solid weighs 8 lbs., and the vessel 6 lbs., the centre of gravity of which is 4 inches from its base.

36. A cylindrical vessel, weighing 8 lbs., will hold 12 lbs. of water. If the centre of gravity of the vessel when empty is 6 inches from the bottom, and when filled with water the centre of gravity of the vessel and contents is raised 3 inches; find the depth of the vessel.

37. From a square whose side is 8 inches, a corner square whose side is 3 inches is taken away; find centre of gravity of the remainder.

38. From a rectangle 4 ft. long, 3 ft. wide, a corner rectangle is cut out, 8 by 6 inches; find centre of gravity of the remainder.

39. From a square whose side is 16 inches, a portion is cut off by a line joining the middle points of two adjacent sides; find centre of gravity of the remainder.

40. From a circular plate whose radius is 8 inches, a circular plate whose radius is 4 inches is cut away, the distance between the two centres is 2 inches; find the centre of gravity of the remainder.

41. From a uniform circular disc whose diameter is 10 inches, another disc having for its diameter the radius of the first circle is cut away; find centre of gravity of the remainder.

42. From a circular disc whose diameter is D, a circular disc whose diameter is d is cut away; find the centre of gravity of the remainder, the distance between their centres being a.

43. From a square whose side is 3 ft., one of the triangles formed by the diagonals is taken away; find the centre of gravity of the remainder.

CHAPTER X.

SIMPLE MACHINES OR MECHANICAL POWERS.

45. Any contrivance which enables us to communicate, change, or prevent motion in a body, may be called a **machine**.

46. The simplest machines, also termed mechanical powers, are :—

1. The Lever.
2. The Wheel and Axle.
3. The Pulley.
4. The Inclined Plane.
5. The Wedge.
6. The Screw.

47. The force applied to any of these machines is called the Power, the force **exerted, resisted,** or **overcome** is called the Weight.

48. **Mechanical advantage.**—The ratio of the weight to the power when in equilibrium is called the Mechanical Advantage of the machine; thus if a force of 4 lbs. sustain a weight of 48 lbs., the mechanical advantage is $\frac{W}{P}$ *i.e.* $\frac{48}{4} = 12$ lbs.

49. **The Lever.**—The lever is a rigid rod movable about a fixed point called the fulcrum.

50. Levers are of 3 kinds, according to the relative position of the power, weight, and fulcrum.

51. Levers of the first kind have the fulcrum between the power and the weight, such as balances, scissors, pincers, crow-bar used in prying.

52. Levers of the second kind have the weight between the power and the fulcrum, such as oars of a boat, nut-crackers, a door swinging on its hinges, a wheel-barrow.

53. Levers of the third kind have the power between the fulcrum and the weight, such as the treadle of a lathe, shears, sugar-tongs.

54. **Condition of Equilibrium in the Lever.** There are three forces acting on the lever, the power, the weight and the reaction at the fulcrum. Taking moments about the fulcrum we have the power multiplied by its distance from the fulcrum equal to the weight multiplied by its distance from the fulcrum. Calling the distances of P and W from the fulcrum a and b, we have $Pa = Wb$, or $\frac{W}{P} = \frac{a}{b}$.

From this we see that in levers of the first kind the mechanical advantage is greater, equal or less than unity as a is greater, equal or less than b.

In levers of the second kind the mechanical advantage is always greater than unity, $\because$ a is always greater than b.

In levers of the third kind the mechanical advantage is always less than unity, $\because$ a is always less than b.

55. In the first kind the reaction of the fulcrum is equal to $P + W$. When P and W are vertical.

In the second kind the reaction of the fulcrum is equal to $W - P$.

In the third kind the reaction of the fulcrum is equal to $P - W$.

In other cases the reaction of the fulcrum can be found by the application of the parallelogram of forces.

EXERCISE XIII.

1. A weight of 30 lbs. is suspended at a distance of 4 inches from the fulcrum of a straight lever of the first

kind, where must a power of 6 lbs. be applied so that there may be equilibrium?

2. A straight rod 8 ft. long, attached to a wall by means of a hinge has a weight of 12 lbs. suspended from its extremity; find the force necessary to keep the rod horizontal, applied at a distance of 3 ft. from the hinge.

3. Two bodies weighing 50 lbs. and 6 lbs. balance at the extremities of a lever 4 ft. long; find the position of the fulcrum.

4. A weight of 6 lbs. at the end of a lever is raised by a force just greater than 15 lbs. which acts 8 inches from the fulcrum, which is at the other end; find the length of the lever and the pressure on the fulcrum.

5. A beam 15 ft. long balances at a point 3 ft. from one end; if a weight of 80 lbs. be suspended from the other end it balances at a point 3 ft. from that end; find the weight of the beam.

6. A beam 12 ft. long, weighing 30 lbs. balances at a point 3 ft. from one end. If a weight of 5 lbs. be suspended 2 ft. from this end; find the weight suspended from the other end would balance the beam about its middle point.

7. The arms of a straight lever are each 6 ft. in length; find what force applied at an angle of 30° with the lever from one end will balance a weight of 20 lbs. hung at the other end; and find pressure on the fulcrum.

SECTION II.

THE BALANCE.

56. **Balances** are simply straight or bent levers of the first kind, having pans suspended from one or

both extremities, for the purpose of determining the weight of a substance.

57. The **Common Balance** is a lever of the first kind with equal arms, having pans suspended from both extremities, and having its C. G. a short distance vertically below the point of support.

58. **A Balance** should be constructed so that :—

1. The beam is perfectly horizontal when the weights in the pans are equal.

2, The beam will deviate perceptibly from its horizontal position when the weights differ by a small quantity; this is called **sensibility.**

3. The beam after being disturbed should return quickly to its original position; this is called **stability**.

59. When great accuracy is required sensibility is of more importance than stability; but in weighing heavy articles stability is of the most importance.

60. **False Balances.**—A balance is false when its arms are of unequal lengths. The true weight of a body may be ascertained by means of a false balance :

1. By placing the body in one pan and balancing it by a counterpoise in the other pan; then remove the body and put in known weights until the beam is again horizontal; these known weights is the weight of the body.

2. Let X and Y be the arms of the balance, W the true weight of the body, and suppose it to weigh P lbs. when placed in one scale, and Q lbs when placed in the other.

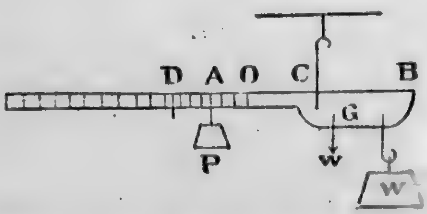
Then by the principle of moments.

$$\begin{aligned} W.x &= P.y, \\ \text{and } W.y &= Q.x, \\ \therefore W^2.xy &= P.Q.xy. \\ W^2 &= PQ, \\ \therefore W &= \sqrt{PQ}. \end{aligned}$$

That is, the true weight is the square root of the apparent weights, when the body is weighed in each scale pan.

61. Of balances with unequal arms, those most commonly used are the common or Roman Steel-yard, the Danish Balance, and the Bent Lever Balance.

62. The **Roman Steel-yard** is a balance with unequal arms. The body to be weighed hangs from the end of one arm, while a movable weight P is made to slide along the other graduated arm, until there is equilibrium.



Let the beam be suspended at C, and let [illegible] weig[illegible] [illegible] P is [illegible] along AC until a point O is found such that the beam is in equilibrium, then

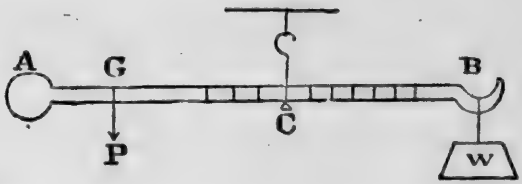
$$P.OC = wCG.$$

A weight of 1 lb. is now hung at B and P is moved along the beam until the point A is found such that the beam is in equilibrium, then

$$P(AO + OC) = wCG + 1.CB,$$
$$\text{but } P.OC = wCG,$$
$$\therefore P.AO = CB.$$

The mark 1 lb. is therefore placed at A, next mark off AD = AO, and place 2 lb. mark at D, &c.

63. The **Danish Balance** consists of a straight bar with a knob at one end. The substance to be weighed is suspended at the other end, the fulcrum C is movable and the bar is so graduated that the position of it determines the weight of the body.



Let P = weight of the beam acting at G, and W the weight of the body suspended at B; then

$$P.GC = W.CB$$
$$\therefore P(GB - CB) = W.CB$$
$$\therefore CB = \frac{P}{P + W}.GB.$$

P. and GB being constant quantities the beam is graduated by taking W equal to 1 lb., 2 lbs., &c., successively.

EXERCISE XIV.

1. A grocer sells tea at the rate of 60 cents per lb., using a balance whose arms are in ratio of 9 : 10; supposing he sells 1 lb. to each of two customers, using first one scale pan and then the other; find whether he gains or loses, and how much per cent.

In one case he would sell $\frac{9}{10}$ of a lb.
And in other " " $\frac{10}{9}$ "
$\therefore$ in two sales he would sell $\frac{9}{10} + \frac{10}{9} = \frac{181}{90}$ lbs. He received pay for 2 lbs., $\therefore$ he loses $\frac{1}{90}$ lb. on a sale of $2\frac{1}{90}$ lbs., $\therefore$ on 100 lbs. he would lose $\frac{1}{90} \times \frac{90}{181} \times \frac{100}{1} = \frac{100}{181}$ = rate per cent. loss.

Since he sold $\frac{181}{90}$ lbs. for $1.20, he sells one pound for $\frac{120}{1} \times \frac{90}{181} = 59\frac{121}{181}$ c.

2. Find the true weight of a body which is found to weigh $6\frac{1}{4}$ lbs. and 4 lbs. when placed in each of the scale pans of a false balance.

3. The arms of a false balance are as 7 : 8. If the true weight of a body is 4 lbs., how much will it appear to weigh in each scale pan?

4. In a common steelyard the weight of the beam is 2 lbs., and the distance of its centre of gravity from the fulcrum is 1 inch.; find where a weight of 3 lbs. must be placed to balance it.

5. 12 lbs. is hung from one end of a Danish Balance, 30 inches long, weighing 2 lbs., which acts at a point 4 inches from one end; find the position of the fulcrum.

6. A body, whose true weight is 8 lbs., appears to weigh 6 lbs. in one scale pan of a false balance; find its weight in the other scale pan.

7. If a balance be false, having its arms in ratio of 15 : 16; find how much per lb. a customer really pays for tea which is sold to him from the shorter arm at 64 cents per lb.

8. One of the arms of a false balance is longer than the other by $\frac{1}{5}$ of the shorter, supposing the scale pans to be used alternately in weighing; find the gain or loss per cent. to the purchaser.

9. The beam of a false balance is 2 ft. 6 in. in length; a body weighs $8\frac{1}{6}$ lbs. in one scale, and 6 lbs. in the other; find its true weight and the length of the arms.

10. A balance has equal arms, but one scale pan is heavier than the other. A certain body weighs 12 lbs. in one pan, and 10 lbs. in the other; find its true weight.

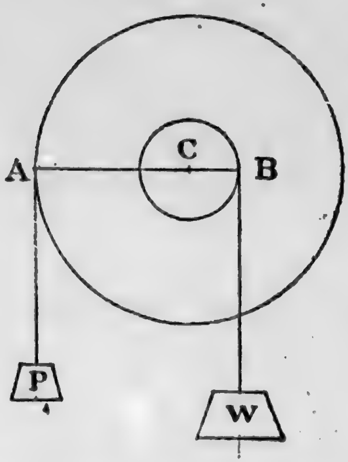
11. A grocer uses a false balance whose arms are as 9 : 10. Supposing he buys tea at 50 cts. per lb., and professes to sell it at 70 cts. per lb.; find his gain per cent.

CHAPTER XI.

THE WHEEL AND AXLE.

64. The simplest form of the **wheel** and **axle** consists of two cylinders having a common axis, the larger of which is called the wheel and the smaller the axle. The weight is applied to the axle by a cord wound around its circumference, and the power, to the wheel in a similar way, having the cord wound in the opposite direction.

Other forms of the wheel and axle are the windlass, capstan, and toothed wheels.



Let the figure represent a cross section of the wheel and axle, C a point in their common axis, AC the radius of the wheel and BC the radius of the axle. P and W may be regarded as two parallel forces, and if in equilibrium their resultant must pass through C, and $\therefore P.AC = W.BC$.

Therefore in the wheel and axle P : W : : radius of axle : radius of wheel.

As the power and weight always act at the same distance from C, the wheel and axle is sometimes called the perpetual lever.

If the thickness of the cord be taken into account, one-half their thickness must be added to their corresponding radius.

EXERCISE XV.

1. If the radii of the wheel and axle are 3 ft. and 6 ins. respectively ; find what weight 12 lbs. will raise.

2. The radius of the wheel is 7 times that of the axle ; find the force necessary to raise a weight of 84 lbs.

3. The radius of the wheel is 18 inches, the diameter of the axle 4 ins., and the cord around the wheel will

only stand a tension of 30 lbs.; find the greatest weight that can be raised.

4. Six sailors work a capstan by means of levers, each exerting a force of 60 lbs., and walk 18 ft. round for every 3 ft. of rope pulled in. What weight can they sustain?

5. A man whose weight is 160 lbs. is just able to support a weight by means of a wheel and axle, whose circumferences are as 2 : 15; find the weight.

6. If the difference between the diameters of the wheel and axle is 7 times the radius of the axle; find the force necessary to sustain a weight of 810 lbs.

7. A weight of 12 lbs. is supported on a wheel and axle, the radii being 30 and 12 inches respectively; find what weight would be supported by the same power, if the radius of each was shortened by 3 inches.

CHAPTER XII.

THE PULLEY.

65. The **Pulley** consists of a small wheel which moves freely about an axis, fixed into a frame-work called the block, and allows a cord to pass over a groove on its circumference.

It is called a fixed pulley when the axis of the pulley is fixed; a movable pulley when the axis is movable.

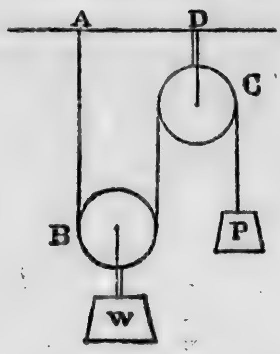
66. **Fixed Pulley.**—No mechanical advantage is gained by a fixed pulley, for as the tension is the

same in every part of the cord, the power must equal the weight.

The effect of a fixed pulley is simply to change the direction of a force.

A fixed pulley can be advantageously used when it is is required to change a pushing into a pulling force; also to raise a small weight to a considerable height, as a man has not to carry his own weight in addition to the weight raised.

67. **The Single Movable Pulley.**



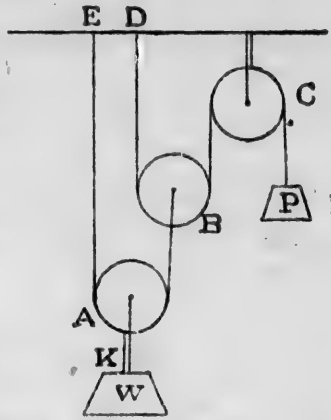
Let W be attached to the movable pulley B, and let B be sustained by a cord ABC, one extremity of which is fastened at A, and the other, passing over the fixed pulley C sustains the power P. The weight W is supported by the tension in BA, and the tension in B. C, $\therefore$ W = sum of these tensions. But the tension of the cord throughout is equal to P,

$$\therefore W = 2P.$$

If the weight of the pulley B, be taken into account, let it be w, then $W + w = 2P$.

68. **The First System of Pulleys.**—In the first system of pulleys, each pulley hangs by a separate cord, each cord has one end fastened to a fixed

beam and all, except the first, have the other end fastened to a movable pulley, as in figure.



Let A B be two movable pulleys and P and W in equilibrium. The tension in the string DCB is P throughout, in the string EAB it is 2P throughout, and the double tension of 2P supports W $\therefore$ W $= 4P$ or 2^2P.

In the single movable pulley $W = 2P$. And when there are two movable pulleys $W = 2^2P$. And in the same manner it may be shewn that when there are three movable pulleys $W = 2^3P$. And hence if there are n movable pulleys,

$$W = 2^nP.$$

If the weight of the movable pulleys be taken into account, let each be w, then the tension of BAE $= 2P - w$, and the tension of AK $= 4P - 2w - w = 4P - 3w = W$,

$$\therefore W + (2^2 - 1)\,w = 2^2P.$$

And similarly if n be the number of movable pulleys,

$$W + (2^n - 1)\,w = 2^nP.$$

EXERCISE XVI.

1. What weight will a power of 4 lbs. support in the first system of pulleys there being 4 pulleys?

$$W = 2^4 .\ 4 = 64 \text{ lbs.}$$

2. In the first system if a power of 3 lbs. supports a weight of 96 lbs.; find the number of pulleys.

$$W = 2^n.P.$$
$$96 = 2^n.3.$$
$$32 = 2^n.$$
$$2^5 = 2^n \therefore n = 5.$$

3. In a system of 4 pulleys, in which each hangs by a separate cord; find the weight supported by a power of 25 lbs.

4. How many pulleys must be employed in the first system, so that 836 lbs. may be raised by 30 lbs.; each pulley weighing 4 lbs?

5. A man weighing 160 lbs. raises a weight of 90 lbs. by means of a fixed pulley; find his pressure on the ground.

6. A man weighing 140 lbs. is suspended from a single movable pulley, and supports himself by pulling on the other end of the string; find the force with which he pulls.

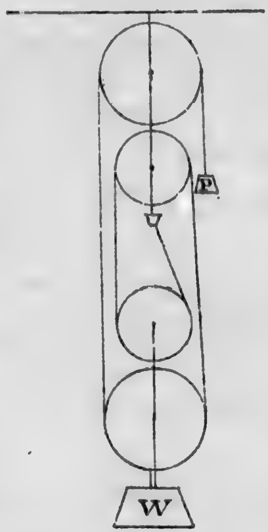
7. In Ex. 6, supposing there were 3 movable pulleys each weighing 3 lbs.; find the force with which he pulls.

8. In the first system of pulleys a power of 10 lbs. supports a weight of 100 lbs. by means of 4 movable pulleys; find the weight of each pulley, supposing their weights equal.

9. In a system of 5 movable pulleys, each hanging by a separate cord, a power of 15 lbs. supports a certain weight; supposing the pulleys to weigh 2, 3, 4, 5 and 6 lbs. respectively beginning with the lowest; find the weight supported.

69. **The Second System of Pulleys.**—In the second system of pulleys there are two blocks of pul-

leys, one of which is fixed to the beam and the other supports the weight. There is only one cord, which passes around a pulley in the upper and lower block alternately, one end of which is fixed to one of the blocks.



Since there is but one cord, and P is attached to one extremity, the tension in every part is equal to P, therefore if there be 4 portions of the cord in contact with the lower block as in fig.,

$$W = 4P.$$

And if there are n cords at the lower block,

$$W = nP.$$

If the weight of the lower block be ω, the weight supported is $W + \omega$, $\therefore W + \omega = nP$.

EXERCISE XVII.

1. What force is necessary to raise a weight of 96 lbs. by an arrangement of six pulleys, the same cord passing around each pulley?

2. In Ex. 1, if the weight of the lower block is 3 lbs.; what force is required?

3. In the second system of pulleys the upper block contains 3 pulleys, and the cord is attached to the lower block; find the weight supported by a power of 4 lbs.

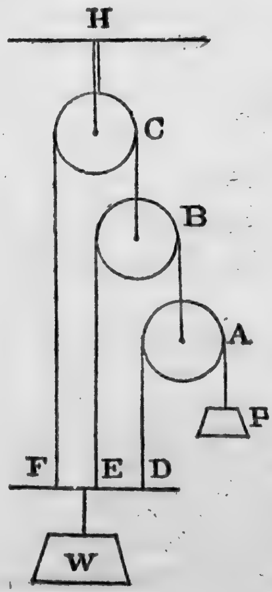
4. Prove that the number of strings at the lower block is always equal to the total number of pulleys in both blocks.

5. In the second system of pulleys if the power is 8 lbs. and the weight 72 lbs., draw the diagram.

6. A man whose weight is 160 lbs., supports himself on the lower block by means of 7 pulleys arranged according to the second system; find the force with which he pulls one end of the rope.

7. If the cord is fastened to the upper block and the weight of the lower block is 8 lbs.; find the weight supported by a power of 12 lbs., there being 3 pulleys in the lower block.

70. **The Third System of Pulleys.**—In the third system of pulleys one end of each cord is attached to the bar from which the weight hangs, and the other supports a pulley (except the cord to which the power is attached). It is simply the first system inverted.



The tension of AD is P; of ABE, 2P; of BCF, 4P; and of CH, 8P. But the weight supported by the beam is W + P.

$$\therefore W + P = 8P,$$

$$i.e.\ W = 2^3P - P.$$

And in the same manner if there were 4 pulleys,

$$W = 2^4P - P.$$

Therefore, when there are n pulleys,

$$W = 2^nP - P = P\,(2^n - 1).$$

If the weight of the pulleys is taken into account, we have the

$$\begin{aligned} &\text{tension in the cord AD} = P, \\ &\text{“} \quad \text{“} \quad \text{“} \quad BF = 2P + w_1, \\ &\text{“} \quad \text{“} \quad \text{“} \quad CF = 4P + 2w_1 + w_2, \end{aligned}$$

and the sum of these equals $W = 7P + 3w_1, + w_2$.

In this system the weights of the movable pulleys assists P.

EXERCISE XVIII.

1. If there are 4 pulleys in the third system, and the power is five lbs.; find the weight.

2. In a system of pulleys in which each cord is attached to the weight a power of 7 lbs., supports a weight of 217 lbs.; find the number of pulleys.

3 If there are 7 pulleys in the third system, and the weight is 254 lbs.; find the power.

4. In the third system of pulleys a weight of 398 lbs. is supported by means of 4 pulleys weighing 3, 4, 5 and 6 lbs., respectively, beginning with the lowest; find the power.

5. If there are 5 pulleys arranged according to the third system weighing 4, 5, 6, 7 and 8 lbs., beginning with the lowest; find the weight supported by a power of 10 lbs.

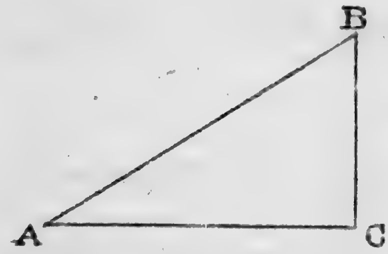
6. A power of 15 lbs. supports a weight of 595 lbs. by means of 5 pulleys of equal weight, arranged according to the third system; find the weight of each pulley.

7. In a system of pulleys in which all the cords are attached to the weight, a weight of 753 lbs. is supported by the pulleys alone, there being 9 pulleys of equal weight; find the weight of each.

CHAPTER XIII.

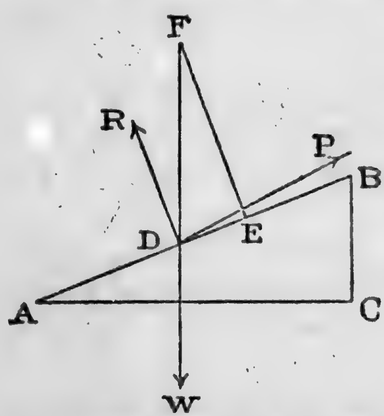
THE INCLINED PLAIN.

71. The **Inclined Plane** is a perfectly smooth, and rigid plane inclined at any angle to the horizon.



Let ABC represent an inclined plane. AB is called the **length**, BC the **height**, and AC the **base**, and BAC the **inclination** of the plane.

72. **Equilibrium** on an inclined plane when the ower acts parallel to the plane.



Let W be a weight resting upon an inclined plane ABC, at the point D, and supported by a power P acting parallel to the plane. The reaction R will be at right angles to the plane.

Cut off DE = BC, and from E erect perpendicucular EF, to meet the direction of W produced upwards in F. The triangle DEF has its sides parallel to the treee forces P, R and W.

$\because \angle EDF = \angle ABC$ (Euc. I., 29) and $\angle DEF = \angle ACB$ each being a right angle, and the side DE =

$BC \therefore DF = AB$, and $EF = AC$ (Euc. I., 26). But by the triangle of forces,

$$\frac{P}{W} = \frac{DE}{DF} = \frac{BC}{AB} = \frac{h}{l}.$$

$$\text{and } \frac{P}{R} = \frac{DE}{EF} = \frac{BC}{AC} = \frac{h}{b}.$$

$$\therefore P : W : R :: h : l : b.$$

EXERCISE XIX.

1. A weight of 15 lbs. is supported on an inclined plane by a power of 9 lbs.; find the reaction of the plane.

$$\frac{P}{W} = \frac{h}{l} = \frac{9}{15} = \frac{3}{5}.$$

$\therefore$ if $BC = 3$, then AB must $= 5$, and $AC^2 = AB^2 - BC^2 = 16$, $\therefore AC = 4$.

$$\text{Again } \frac{R}{P} = \frac{b}{h}$$

$$\frac{R}{9} = \frac{4}{3} \quad \therefore R = 12 \text{ lbs.}$$

2. If $l = 13$, $h = 5$, and $W = 65$; find P and R.

3. If $P = 14$, $h = 7$, and $b = 24$; find W and R.

4. If $P = 18$, $W = 82$, and $h + b = 49$; find R and l.

5. If $P + W = 216$, $h = 11$, and $b = 60$; find P., W, and R.

6. If $W - P = 72$, $b = 84$, and $l = 85$; find P., W, and R.

7. If $P = \frac{W}{2}$; find the inclination of the plane.

8. A plane is inclined at an angle of 60°; find the force acting along the plane necessary to support a weight of 36 lbs. on the plane.

9. What power, acting parallel to the plane, will support a weight of 78 lbs. on a plane rising 5 ft. in 13 ft.?

10. The length of an inclined plane is 17 ft., and the height 8 ft.; find into what two parts a weight of 200 lbs. must be divided so that one part hanging over the top of the plane may balance the other part resting on the plane.

11. Two weights connected by a cord passing over a pulley at the summit of an inclined plane, on which one weight rests, and for every 11 inches that the one is made to descend, the other rises 3 inches; find the ratio of the weights.

12. Two planes of the same height are placed back to back, and two weights of 5 lbs. and 13 lbs., connected by a string passing over their common vertex, balance each other on the planes; find the ratio of the lengths of the planes.

13. If it takes 3 times the power to support a given weight on an inclined plane, ABC, when placed on the side AC, that it does when placed on the side BC; find the greatest weight that a power of 5 lbs. can support on the plane.

14. If a weight of 50 lbs. is supported on a plane, by means of a cord fastened to a point on the plane, which will only bear a tension of 25 lbs.; the plane is gradually tilted; what is the greatest angle it can have so as not to break the cord?

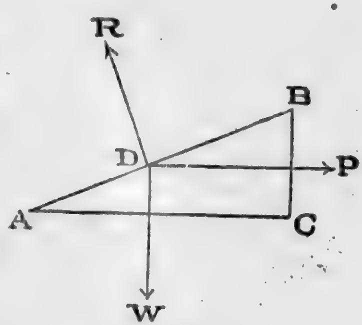
15. A man on roller skates supports himself on an inclined plane rising 4 ft. in 15 ft., by means of a rope

which is parallel to the plane; find the tension in the rope, supposing the man weighs 152 lbs.

SECTION II.

73. Equilibrium on an **Inclined Plane** when the power acts parallel to the base.

Let W be a weight resting upon an inclined plane ABC, at the point D, and supported by a power P., acting parallel to the base. The reaction R will be at right angles to the plane.



$\because$ P acts at right angles to BC
and W " " " AC
and R " " " AB

$\therefore$ by the triangle of forces if P, W and R are in equilibrium, they must be proportional to the sides of the triangle ABC,

$$i.e.\ P : W : R :: BC : AC : AB,$$

$$\text{or } \frac{P}{W} = \frac{h}{b},\ \frac{P}{R} = \frac{h}{l} \text{ and } \frac{R}{W} = \frac{l}{b}.$$

EXERCISE XX.

1. Find the weight supported on an inclined plane whose height is 5 ft. and length 13 ft. by a power of 20 lbs. acting parallel to the base.

2. If $P = 16$, $W = 30$, and $h = 8$; find b, l and R.

3. If $h = 12$, $l = 37$, $W = 70$; find P and R.

4. If $P = 7$, $W = 24$; find R, and ratio of h, b and l.

5. If $R = 130$, $W = 126$, and $h = 16$; find P, b and l.

6. If $P = 36$, $R = 164$, and $b = 40$; find W, h and l.

7. If $P + W = 291$, $h = 13$, $l = 85$; find P, W and R.

8. If a force of 90 lbs. acting horizontally supports a weight of 400 lbs. on an inclined plane; find the force acting parallel to the plane necessary to sustain the weight.

9. A weight of P is supported upon an inclined plane by a power of P acting horizontally; find the inclination of the plane.

10. Find the force acting horizontally that will sustain a weight of 40 lbs. on a plane whose inclination is 30°.

11. If a power of 2P acting horizontally support a certain weight on an inclined plane, and a power of P acting parallel to the plane would support the same weight on the plane; find the inclination of the plane.

12. Find the weight supported on an inclined plane rising 16 in 65, by two forces, one of 8 lbs. acting parallel to the plane, and the other of 12 lbs. acting horizontally.

13. If the pressure on the plane is 12 lbs. when the power acts horizontally and 9 lbs. when it acts parallel to the plane; find the weight.

14. A power P acting along a plane can support a weight of 13 lbs. and acting horizontally can support a weight of 12 lbs.; find P.

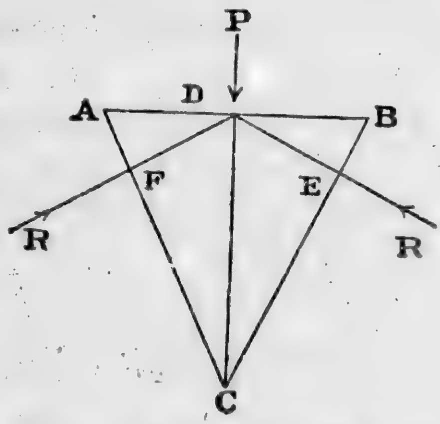
15. A weight W can be supported by a force of 14 lbs. acting parallel to the plane, or by a force of $14\frac{7}{12}$ lbs. acting horizontally; find W.

CHAPTER XIV.

THE WEDGE.

74. The wedge is a double inclined plane, used for separating bodies. The power is applied in a direction perpendicular to the height of the plane *i.e.* parallel to the base.

The resistance acts in a direction at right angles to the inclined surface of the wedge *i.e.* the length of the plane.



Let ABC be an isosceles wedge, and E, F two points similarly situated. The pressure on each side of the wedge will be the same, suppose R. Let a power P act at D the middle point of AB. Then by theory of inclined plane, when P acts parallel to the base, $\frac{P}{R} = \frac{AD}{AC} = \frac{\frac{1}{2}\text{ back of wedge}}{\text{length of a side}}$.

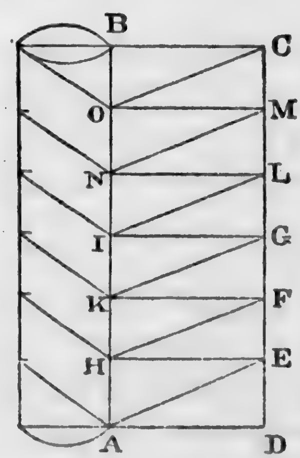
75. The force required to split or separate the particles of a body is generally so great that instead of applying a pushing force, a series of blows is imparted to it. Therefore the theory of the wedge is of very little importance.

CHAPTER XV.

THE SCREW.

76. The **Screw** is a combination of the lever and inclined plain. It consists of a right cylinder with a uniform projecting thread around its surface, inclined at a constant angle to the axis of the cylinder. This cylinder works in a concave cylinder, called the nut, having a spiral cavity on its surface corresponding to the thread of the screw.

The screw is sometimes used to overcome resistance and sometimes to increase pressure.



Let AB be a cylinder, AC a rectangle whose base AD is equal to the circumference of the cylinder. Let CD be divided into any number of equal parts, and AB into the same number of equal parts; join AE, HF, KG, IL, etc.

Then if the rectangle be wrapped around the cylinder the lines AE, HF, etc., will represent the threads of the screw, the distance between two contiguous threads being DE. Thus the screw may be regarded as an inclined plane wrapped around a cylinder; the resistance to be overcome as a weight resting on it. The height of the plane being the distance between two contiguous threads and the base the circumference of the cylinder.

The power is applied at right angles to the axis of the cylinder, and therefore is parallel to the base of the inclined plane forming the screw. Then if the power acts at the circumference of the cylinder we have,

$$\frac{P}{W} = \frac{h}{b} = \frac{\text{distance between contiguous threads}}{\text{circumference of cylinder}}.$$

But since the power is generally applied by means of a lever; by the principle of moment we have the power acting at the circumference of the cylinder is to the power acting at the end of the lever as the radius of the cylinder is to the arm of the power (or length of the lever),

$$\text{or} \quad P : P^1 :: r : a,$$

$$\therefore P = P^1 \frac{r}{a}.$$

Supplying this value of P in former equation and dividing by $\frac{r}{a}$ we have,

$$\frac{P}{W} = \frac{\text{distance between contiguous threads}}{2\pi a},$$

$$\therefore \frac{P}{W} = \frac{\text{distance between contiguous threads}}{\text{circumference described by the power}}.$$

EXERCISE XXI.

1. The diameter of a screw is 2 inches, and the distance between the threads $\frac{1}{3}$ of an inch; find the resistance overcome by means of a power of 10 lbs. applied at the end of a lever 9 inches in length.

2. Find the mechanical advantage in a screw having

7 threads to an inch, the radius of the cylinder being $\frac{1}{2}$ in. and the arm of the power 14 inches.

3. The arm of the power is $10\frac{1}{2}$ ins., and the distance between the threads $\frac{1}{4}$ of an inch; find the power necessary to produce a pressure of 15 cwt.

4. What must be the distance between the threads of a screw so that a power of 7 lbs. applied at the end of a lever 10 inches long may sustain a weight of 500 lbs.?

5. Find the power applied at the end of a lever 21 inches in length to raise a weight of 630 lbs., by means of a screw, having 8 threads to an inch.

6. If a power of 9 lbs. describe a revolution of 5 ft., while the screw moves through $\frac{1}{3}$ of an inch; find the pressure produced.

7. If the angle of the thread of a screw be 30°, and the length of the arm of the power 6 times the radius of the cylinder; find the weight supported by a power of 8 lbs.

8. Find the mechanical advantage in a screw having 5 threads to an inch, the diameter of the circle described by the power being 28 inches.

9. A power of 21 lbs. produces a pressure of 1,540 lbs. by means of a screw having 4 threads to an inch; find the length of the power-arm.

CHAPTER XVI.

VIRTUAL VELOCITIES AS APPLIED TO MACHINES.

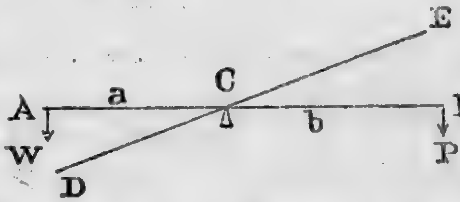
77. If a machine is in equilibrium, and we suppose it to receive any displacement, consistent with the connection of its various parts, then the spaces des-

cribed by the power and weight, estimated in their respective directions, are called the **Virtual Velocities** of the power and weight.

78. **Principle** of Virtual Velocities.—In any machine the power multiplied by its virtual velocity is equal to the weight multiplied by its virtual velocity.

79. The principle of virtual velocities may be shown to be true for all the mechanical powers as follows:—

80. **The Lever.** Let AB represent the lever, whose fulcrum is C, the weight W acting at A, and the power P at B. The arms being a and b respectively. Suppose the lever to receive any displacement, so as to occupy position DCE. Let the angle ACD be n degrees; then the weight has moved through the distance $2\pi a\frac{n}{360}$; and the power has moved through space $2\pi b\frac{n}{360}$.



Then by principle of virtual velocities

$$W.2\pi a\tfrac{n}{360} = P.2\pi b\tfrac{n}{360}$$
$$i.e.\ W.a = Pb,$$

which is the condition of equilibrium in the lever as shewn in § 54.

81. **The Wheel and Axle.**—Suppose the wheel and axle to make a complete revolution, then W has moved through distance $2\pi r$, and P through $2\pi R$.

$$\therefore P.2\pi R = W.2\pi r$$
$$i.e.\ P.R = W.r,$$

the result obtained in § 64.

82. **The Pulley.**—In the figure, § 68, let W be raised 1 inch, the string on each side of the pulley A will be shortened 1 inch, therefore the pulley B must raise 2 inches, and the string on each side of B be shortened 2 inches, ∴ P descends through 4 inches.

$$\therefore 4P = W.$$

In same manner, if there were 3 movable pulleys, we would have

$$8P \text{ or } 2^3P = W,$$

and if n pulleys $2^nP = W,$

the result obtained in § 68.

In figure 69, let W be raised 1 inch, if there were n strings between the two blocks, each of them will be shortened 1 inch; therefore P will descend through n inches.

$$\therefore nP = W,$$

the result obtained in § 69.

In figure, § 70, let W be raised 1 inch, then pulley B will descend 1 inch, and pulley C through $(2+1)$ inches, and P through $[2(2+1)+1]$ inches, or (2^3-1) inches; in same manner if there were 4 pulleys P would descend through (2^4-1) inches, and if n pulleys through (2^n-1) inches.

$$\therefore (2^n-1)\,P = W,$$

the result obtained in § 69.

83. **The Inclined Plane.**—In figure, § 72, let W be drawn from A to B; then P must move down through a distance AB, and W is raised the height of the plane or BC.

$$\therefore P.AB = W.BC,$$

the result obtained in § 72.

In figure, § 73, let W be drawn up from A to B; then P must pass through a distance AC, and W is raised through distance BC.

$$\therefore P.AC = W.BC,$$

$$\frac{P}{W} = \frac{h}{b},$$

the result obtained in § 73.

84. **The Screw.**—Suppose P to make a complete revolution, it will pass through the space $2\pi r$ or circumference of circle described by power, and W will be raised through the distance between contiguous threads. $\therefore$ P × circumference described by power = W × distance between contiguous threads.

EXERCISE XXII.

1. When a power of 8 lbs. is applied to lift a weight of 265 lbs. it descends through 53 inches; find how far the weight is raised.

2. A power passing through 11 ft. raises a weight of 627 lbs. through 4 inches; find the power.

3. By means of a lever a power of 30 lbs. passing through 3 inches raises a weight 1 inch; find the weight.

4. In the wheel and axle a certain power supports a weight of 880 lbs.; when the power descends through 6 ft. the weight is raised 9 inches; find the power.

5. In a wheel and axle the radius of the wheel is 13 times that of the axle, a power P balances a weight W; find how high the weight must be raised so that the power may descend through 14 ft. more than the weight is raised.

6. In the first system of pulleys it is found that when P and W are in equilibrium, the power descends through 32 ft. for every foot the weight is raised; find the number of pulleys.

7. In the second system of pulleys if P descends through 15 ft. while W. rises through 3 ft.; find the number of strings at the lower block.

8. A screw takes 48 turns to pass through 2 ft., the circumference described by the power is 6 ft.; find the ratio of P to W.

9. Two inclined planes of the same height, slope in opposite directions, and two weights rest, one on each plane, connected by a cord passing over a pulley at the common vertex of the planes. If the lengths of the planes are 6 ft. and 9 ft.; find the relation of the weights.

MISCELLANEOUS QUESTIONS.

(Taken from various Examination Papers,)

1. How are statical forces measured?

2. State the principle of the transmissibility of force. By what experiments could this principle be illustrated (1) for pressures, (2) for tensions.

3. Enunciate the triangle of forces explaining your enunciation by means of a diagram in which the directions of action of the forces are marked by arrows. Mark also the point of application of the forces. Shew that *perpendicular* may be substituted for *parallel* in the enunciation.

4. Shew how to find the resultant of three given forces acting on a point; and prove that to produce equilibrium their directions must lie in the same plane.

5. Shew that as the angle between two forces is increased their resultant is diminished.

6. Define the resultant of any number of forces. If a system of forces be in equilibrium, prove that each of these forces is equal to the resultant of all the rest, and acts in a direction directly opposite to the direction of that resultant.

7. Indicate by a drawing the forces that keep a kite in equilibrium in the air.

8. By aid of a sketch explain the resolution of forces in the case of a ship sailing at right angles to the wind,—exhibiting also the force that causes lee-way.

9. State the condition in order that three or more forces acting on a bar, which is free to turn about a fixed axis, may not produce motion about that axis.

10. If two forces acting on a point are represented in magnitude and direction by two sides of a triangle, under what circumstances will the third side correctly represent their resultant?

11. What is meant by the moment of a force about a given point? How is its magnitude determined?

12. The moment of a given force about a given point is the same, no matter at what point in its line of action the force is supposed to act.

13. The sum of the moments of two forces with respect to any point in their plane is equal to the moment of their resultant with respect to the same point.

14. What is meant by the moment of a force? How can moments be represented if forces are represented by straight lines?

15. How can the centre of gravity of a body be determined experimentally?

16. State the principle of virtual velocities. Define the term virtual velocity.

17. Deduce the parallelogram of forces from the principle of virtual velocities.

18. Given the centres of gravity of a body and any part of it, show how to find the centre of gravity of the remainder.

19. From a rectangle 6 inches wide, there is cut an isosceles triangle having one of the longer sides of the rectangle for its base, the centre of gravity of the remain-

ing piece of the rectangle is at the summit of the triangle; find the height of the triangle.

20. What are the conditions that two forces acting on a body may produce no effect? What are the conditions for three forces so acting?

21. Show how to arrange pressures of 3 lbs., 4 lbs., 12 lbs. and 13 lbs. to produce equilibrium.

22. How can a statical force be represented by a circle? In what respects can it be represented by a sphere?

23. State the law of the tensions of the parts of a perfectly flexible weightless cord passing around one or more smooth pulleys.

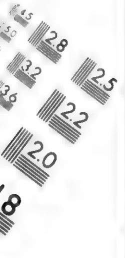
24. Prove that if S be the centre of the circle circumscribing the triangle ABC, and P the point of intersection of the perpendiculars from the angles on the opposite sides, the resultant of forces represented by SA, SB, SC, will be represented by SP.

25. Examine the truth of the following statement:—"If three forces acting on a body are parallel to the sides of a triangle they will keep it at rest."

26. Three forces, the first acting at the point A, the second at the point B, the third at the point C, are represented in magnitude and direction by the lines AB, BC, CA. Determine the resultant.

27. Is the weight of a body the same at all points of the earth's surface? How could any difference be detected?

28. Show that as the angle between the forces is increased the resultant is diminished. If each force be increased by a force of the same magnitude; how will


4.5
5.0
2.8
3.2
2.5
3.6
2.2
2.0
1.8

1.0

the direction of the resultant be affected ; and how if the forces be increased in the same ratio ?

29. Define a couple, and shew that the forces composing one do not admit of a single resultant.

30. State the various transformations that may be made on a couple without alteration of effect. Establish the truth of one of them.

31. The sides of a quadrilateral are acted on by forces perpendicular to them, and proportional to them in magnitude, the forces being turned inwards. Shew that if the points of application divide the sides in a constant ratio they reduce to a couple.

32. Find the centre of gravity of 3 uniform rods forming a triangle. If the system be suspended by a string attached to a point in one of the sides, find the position of the point that the triangle may rest with one side vertical.

33. What are the 3 kinds of statical equilibrium, and how are they connected conditionally with the centre of gravity ?

34. Where is the centre of gravity of a triangle situated ?

35. Under what conditions will two forces not have a resultant ?

36. A weight of 100 lbs. is suspended by a rope 20 ft. long ; what horizontal force is required to draw it 6 ft. out of vertical line ?

37. A right-angled triangle whose sides are 3, 4 and 5, and whose unit of surface weighs 1 lb. is suspended by a cord attached to the middle point of the longest side,

what weight must be placed upon the acute angle to keep the long side horizontal?

38. Distinguish between mass and weight, density and specific gravity.

39. What is the unit of force commonly adopted in statics? What general relation is there between the latitude of any place and the magnitude of the statical unit of force for that place?

40. What is meant by saying that two or more given forces exactly balance each other?

41. If a body moving with a constant velocity in a straight line be brought under the action of two forces which exactly balance each other, what will be the result with regard to the motion of the body?

42. Explain how a force may be completely represented by a straight line.

43. Two forces of 10 units each act in lines which meet in a point, and the angle between their directions is 120°, shew that they may be balanced by two forces of 5 units each, and determine the direction in which these must act.

44. Explain what is meant by the statement, "The body A is at rest relative to the body B." Give illustrations.

45. Two forces acting in lines which meet in a point are represented by the straight lines AB, AC; shew that their resultant is represented by 2AD, where D is the point of bisection of the straight line BC.

46. Four forces acting in lines which meet in a point are represented by the straight lines AC, BC, AD, BD;

shew that their resultant is represented by 4EF, where E and F are the respective points of bisection of the diagonals AB, CD of the quadrilateral ACBD.

47. What are the conditions of equilibrium of two forces?

48. What are the conditions of equilibrium of three forces; (*a*) if two of them are parallel to one another, (*b*) if there are two not parallel to one another?

49. A body is pulled N., S., E., and W. by strings whose directions meet in a point, the forces of tensions along the strings being equal to 26, 110, 75, and 88 lbs. respectively. Shew that these forces may be balanced by a force of 85 lbs. in the proper direction, and by no other single force whatever.

50. How is the moment of a force about a given point measured?

51. The quadrilateral ABCD is held in equilibrium by forces which act along the sides AB, AD, CB, CD, and which are proportioned to a, d, b, c times these sides reectively. Shew that $ac = bc$.

52. A downward pressure of 5 lbs. applied at the point P is represented by a straight line an inch and a quarter long drawn from that point from left to right. How would an upward pressure of 7 lbs. applied at P be represented?

53. Shew how to determine the magnitude and the point of application of two forces acting on a rigid body along parallel lines.

54. ABCD is a square. A force of 3 lbs. acts from A towards B; a force of 4 lbs. from B towards C; a force

of 5 lbs. from D towards C; a force of 12 lbs. from A towards D. Determine the magnitude of the resultant.

55. Three parallel forces acting on a rigid body are in equilibrium. Prove that the moment of any one of them is equal and opposite to the algebraic sum of the moments of the other two.

56. What is meant when a line is said to completely represent a force?

59. Find the centre of gravity of five equal weights placed at 5 of the angles of a regular hexagon.

58. Show how to resolve a given force into two components, one of which has a given magnitude and acts parallel to a given straight line.

59. Shew how it is possible for a sailing vessel to make way in a direction different from that of the wind.

60. Why cannot a round tub be steered at as great an angle to the direction of the wind as a long boat?

61. When a horse is employed to tow a barge along a canal the tow-rope is usually of considerable length; give definite reason for using a long rope instead of a short one. Shew whether the same considerations hold good in relation to the length of the rope when a steam-tug is used instead of a horse.

62. If a man wants to help a cart up hill is there any mechanical reason why he should put his shoulder to the wheel, instead of pushing at the body of the cart? And if so, shew at what part of the wheel force can be applied with the greatest effect.

63. A body weighing 6 lbs. is placed on a smooth plane, which is inclined at 30° to the horizon; find the

two directions in which a force equal to the body may act to produce equilibrium. Also find what is the pressure on the plane in each case.

64. A heavy plummet is immersed in a stream, the string being held by a person standing on the bank. The string is found to settle in a sloping position. Shew by means of a sketch the three forces which keep the plummet in equilibrium.

65. The extremities of the horizontal diameter of a circular disc, weighing 6 ozs., are nailed against a wall, and to a point in the edge of the disc at $\frac{1}{12}$ of the whole circumference from one of the nails a weight of 4 ozs. is attached; find the pressure on each nail.

66. The weight of a window sash 3 ft. wide is 5 lbs., each of the weights attached to the cords is 2 lbs.; if one of the cords be broken, find at what distance from the middle of the sash the hand must be placed to raise it with the least effort.

67. Shew that if two forces act upon a body the moment of the one about a point in their resultant is equal to that of the other about the same point.

68. If a right circular cone will just rest on its side upon a table with its vertex projecting over the edge of the table to the distance of $\frac{1}{3}$ of the slant side, find the ratio of the altitude to the radius of the base.

69. A rod is supported by two strings attached to its extremities. One of the strings is fastened to a small ring, and the other passes through the ring and is gradually lengthened; shew that the centre of gravity of the rod will describe a straight line.

70. Explain clearly why a loaded wagon going obliquely across a declivity may turn over, while if the same load could be compressed, the wagon might safely pass the same spot in the same course.

71. A short circular cylinder of wood has a hemispherical end. When placed with its curved end on a smooth table it rests in any position in which it is placed. Determine the position of its centre of gravity.

72. Squares are described upon the three sides of an isosceles right-angled triangle. Determine the centre of gravity of the complete figure so formed.

73. A sphere of wood loaded at one end with lead rests upon a plane inclined at 30° to the horizon, being prevented from sliding down by the friction of the plane. State and explain by a diagram the conditions of equilibrium.

74. A nail in the middle of a square board is pulled equally by two strings which pass over pulleys fixed to two adjacent corners of the board. If T be the tension in each string, find the magnitude and direction of the pressure on the nail.

75. From each end of the base of a uniform triangular disc, a weight equal to that of the disc is hung. Find the point at which the disc must be supported to rest in a horizontal position.

DYNAMICS.

KINEMATICS—MOTION.

CHAPTER I.

1. Forces not in equilibrium acting on a body produce motion. The rapidity with which a body moves is called its velocity. Velocity or rate of motion is measured by the space traversed in a given time. The unit of time is one second; the unit of length is generally one foot, but varies in different countries.

2. **Unit of Velocity.**—A body is said to be moving with a unit of velocity when it moves through unit of space in unit of time.

3. **Uniform Velocity.**—When a body describes equal spaces in equal times the velocity is uniform, or constant.

If a body moves through 120 feet every second for 6 seconds the velocity is uniform, and the space described is $120 \times 6 = 720$ feet; and generally if s denote the space described by a body in t seconds and v denote the velocity, or number of feet described in one second, then

$$S = vt.$$

If a body rotates through an angle of θ in one second, θ is said to be the angular velocity. If a be the measure of the arc described by the body, then

$$S = at.$$

NEWTON'S LAWS OF MOTION.

Newton enunciated three laws of motion which have generally been made the basis of Dynamics.

First Law of Motion.—Every body continues in a state of rest, or of uniform motion in a straight line if not acted on by any external force.

Second Law of Motion.—Change of motion is proportional to the external force applied, and takes place in the direction of the external force. Or, when several forces act simultaneously on a body, each produces the same effect as if it had acted separately.

Third Law of Motion.—Action and reaction are equal and opposite, *i.e.* to every action there is a corresponding reaction equal in magnitude and opposite in direction.

EXAMPLES I.

1. If a body moves through 600 feet in 8 seconds; find its velocity if uniform.

2. A body having a uniform rotatory velocity of 5° in 20 seconds; through what angle will it pass in 3 hours?

3. Through how many degrees will the hour hand of a clock pass in 17 minutes?

4. Supposing the minute hand of a clock is $4\frac{1}{8}$ inches long; how far will the end travel while the hour hand passes through an angle of 15°? ($\pi = 3\frac{1}{7}$.)

5. A regular hexagonal wheel whose side is 1 foot 9 inches, makes 600 revolutions in 3 minutes; find the velocity of one of its angular points.

6. One body starts from A towards B with a uniform velocity of 15 feet per second; another body from B towards A with a uniform velocity of 30 feet per second, where will they meet; the distance from A to B being 60 yards?

7. Two bodies start at the same moment in the same direction from two points 800 feet apart, when will they be together supposing the first has a uniform velocity of 10 ft. per second and the other one of $9\frac{1}{2}$ ft. per second?

8. Can a body remain at rest when acted upon by velocities 60 ft., 23 ft. and 27 ft. respectively?

CHAPTER II.

4. **Variable Velocity.**—When the velocity of a body increases or decreases uniformly, the motion is said to be uniformly accelerated or retarded.

5. Uniform acceleration or retardation is measured by the increase or decrease of the velocity per second. Thus, if a body had a velocity of 8 ft., 12 ft., and 16 ft., in three successive seconds, it is said to have a uniform acceleration of 4 ft. per second. Or if the body started with a velocity of 25 ft. per second, and at the end of the first second it only had a velocity of 20 ft., and at the end of the second second a velocity of 15 ft., it is said to have a minus acceleration, or a retardation of 5 ft. per second.

If f represent the acceleration or retardation of a moving body,

The velocity	gained	or lost	in	1	sec.	is	f.
"	"	"	"	"	2	"	$2f$.
"	"	"	"	"	3	"	$3f$.
"	"	"	"	"	t	"	tf.

$$\therefore v = tf.$$

6. Examples: (1) A body starting from a rest has acquired a velocity of 45 miles per hour during 10 minutes; find the acceleration of the body in feet per second.

$$\text{velocity} = \frac{45 \times 1760 \times 3}{60 \times 50} \text{ ft. per sec.} = 66.$$

$$v = tf$$
$$\therefore 66 = 10 \times 60 \times f,$$
$$\therefore f = \tfrac{11}{100} \text{ ft. per sec.}$$

(2) With what velocity must a body start so as to come to rest in 10 seconds if its velocity be retarded 8 ft. per sec.?

$$v = tf$$
$$v = 10 \times 8 = 80.$$

The velocity lost is 80 ft. per sec., ∴ it must have started with a velocity of 80 ft. per sec.

7. **Space Described by a Body Uniformly Accelerated.**—Suppose a body to pass through 5 miles the first hour, through 10 miles the second hour, and through 15 miles the third hour, the space described in the 3 hours $= 5 + 10 + 15 = 30$ miles, or it has a mean velocity of 10 miles per hour, during the 3 hours. The initial velocity was 5 and

the terminal velocity 15, $\therefore$ space $= 3\,\frac{5+15}{2}$ or $S = t \times$ mean of initial and terminal velocity.

And since terminal velocity $= tf +$ initial velocity; taking V for initial velocity we have

$$S = t\left(\frac{V + tf + V}{2}\right) = t\left(V + \frac{tf}{2}\right)$$

or $S = t\left(\frac{V+v}{2}\right) \because v = tf =$ vel. at the end of t secs. $+$ V.

If the initial vel. or $V = 0$,

$$\text{then } S = \frac{t^2 f}{2} = \frac{tv}{2}$$

8. To find the space described in any particular second when a body is uniformly accelerated.

If the body start from rest the space described in first second is $\frac{f}{2}$,

$$\because S = \frac{t^2 f}{2} \text{ but } t \text{ is } 1 \;\therefore S = \frac{f}{2}$$

Space described in 2 secs. $= \frac{4f}{2}$ $\therefore$ space described in 2nd sec. $2f - \frac{f}{2} = \frac{f}{2}\,(2,\,2 - 1)$.

Space described in 3 secs. $= \frac{9f}{2}$ $\therefore$ space described in 3rd sec. $= \frac{9}{2}f - 2f = \frac{f}{2}\,(2,\,3 - 1)$.

Space described in 4 secs. $= \frac{16f}{2}$ $\therefore$ space described in 4th sec. $= 8f - \frac{9}{2}f = \frac{f}{2}\,(2,\,4 - 1)$.

$\therefore$ space described in t^{th} sec. $= \frac{f}{2}\,(2t - 1)$,

$$\text{or } S = (2t - 1)\frac{f}{2}.$$

9. We have found that $v = tf \therefore t = \dfrac{v}{f}$

$$\text{and that } S = \frac{t^2 f}{2} \therefore S = \frac{v^2}{f^2} \cdot \frac{f}{2} = \frac{v^2}{2f},$$

$$\therefore v^2 = 2fS.$$

EXAMPLES.

1. Through what space will a body pass in 6 sec. under an acceleration of 8 ft. per sec. if it commences to move with a velocity of 22 feet per sec.?

$$V = 22, \quad v = 22 + 6 \times 8 = 70.$$

$$S = t\left(\frac{V + v}{2}\right)$$

$$S = 6\left(\frac{22 + 70}{2}\right) = 276 \text{ ft.}$$

2. A body passes through 225 ft. in 5 secs., starting with a vel. of 15 ft.; find its acceleration.

$$S = t\left(V + \frac{tf}{2}\right)$$

$$225 = 5\left(15 + \frac{5f}{2}\right)$$

$$45 - 15 = \frac{5f}{2})$$

$$\therefore f = 12 \text{ ft. per sec.}$$

3. A body is observed to pass through 28 ft. and 40 ft. in two successive seconds; find the space it would describe in the 8 sec. from rest.

$$S = (2t - 1)\frac{f}{2}$$

$$S = (16 - 1)\frac{12}{2}$$

$$= 90 \text{ ft.}$$

4. Find the velocity of a body which starting from rest with an acceleration 15 ft. per sec., has passed through 45 feet.

$$v^2 = 2fS$$
$$v^2 = 2 \times 15 \times 45$$
$$= 1350$$
$$\therefore v = 15\sqrt{6}.$$

5. Through what space must a body move under an acceleration of 8 ft. per sec., so that its velocity may increase from 12 ft. to 30 ft.

Space described to produce a vel. of 30 ft. $= \frac{30^2}{2 \times 8}$

" " " 12 " $= \frac{12^2}{2 \times 8}$

$\therefore$ " increase fr. 12 to 30 ft. $= \frac{1}{2 \times 8}(30) - 12^2) = 47\frac{1}{4}$ ft.

$$\therefore S = (v^2 - V^2) \div 2f.$$

EXERCISE II.

1. Through what space will a body pass in 8 seconds under an acceleration of 10 ft. per second, having an initial velocity of 20 ft. per second?

2. What space will a body describe in 10 seconds with an acceleration of 20 ft. per second; starting from rest?

3. Find the acceleration of a body starting with a velocity of 20 ft. per second, and describing a space of 300 ft. in 6 seconds.

4. What must be the initial velocity of a body describing 552 ft. in 12 seconds, under an acceleration of 6 ft. per second?

5. In what time will a body having an initial velocity of 12 ft. per second describe a space of 192 ft., under an acceleration of 3 ft. per second?

6. What space will a body describe, moving from rest under an acceleration of 5 ft. per second in 12 seconds?

7. In what time will a body moving from rest describe a space of 270 ft. under an acceleration of 15 ft. per second?

8. What is the velocity of a body which moving from rest with an acceleration of 18 ft. per second, has described 400 ft.?

9. Find the acceleration of a body which moving from rest has passed through 400 ft. in 5 seconds.

10. A body moving from rest has described 288 ft. in 8 seconds; find its velocity.

11. Find the velocity of a body after 6 seconds moving from rest under an acceleration of 12 ft. per second.

12. A body describes 48 ft. before coming at rest under a retardation of 6 ft. per second; find its initial velocity.

13. A body starts with a velocity of 120 ft. and loses a third of its velocity per second; how far will it move?

CHAPTER III.

GRAVITY.

10. **Force of Gravity.**—The accelerating force of gravity has been experimentally determined by means of Atwood's machine, and by vibrations of pendulums to be about 32.2 feet per second. The accelerating force of gravity is generally denoted by g.

When gravity is considerated the formulæ in 7, 8 and 9 become

$$
\begin{aligned}
s &= \tfrac{1}{2}gt^2 = \tfrac{1}{2}vt \\
v &= gt \\
v^2 &= 2gS \\
s &= \frac{v^2 - V^2}{2g} \text{ or } v^2 = V^2 + 2gS.
\end{aligned}
$$

Space described in t^{th} second $= (2t - 1)\frac{g}{2}$. By simply substituting g for f.

EXAMPLES.

1. How long must a stone fall under the action of gravity to acquire a velocity of 161 feet per second?

$$
\begin{aligned}
v &= gt \\
161 &= 32.2t \\
\therefore t &= 5 \text{ secs.}
\end{aligned}
$$

2. A body is thrown down with a velocity of 15 ft. per second from the top of a tower and reaches the ground in 4 seconds; find height of the tower.

$$
\begin{aligned}
S &= t\left(V + \frac{tg}{2}\right) \\
S &= 4\left(15 + \frac{4 \times 32.2}{2}\right) \\
&= 4\,(15 + 64.4) \\
&= 317.6 \text{ ft.}
\end{aligned}
$$

3. A body is thrown vertically upwards with a velocity of 322 ft. per second; find its velocity when it is 150 ft. high.

$$
\begin{aligned}
v^2 &= V^2 - 2gS, \because \text{ gravity is retarding,} \\
v^2 &= (322)^2 - 64.4 \times 150 \\
\therefore v &= 306.6 \text{ ft. per second.}
\end{aligned}
$$

4. A stone dropped from a balloon ascending 30 ft. per second, reaches the ground in 6 seconds ; find the height of the balloon when the stone was dropped.

$$S = t\left(V + \frac{g\,t}{2}\right).$$

But V is in contrary direction to g in this case, ∴ we must regard it as negative.

$$S = 6\left(-30 + \frac{32.2 \times}{2}\right)$$
$$= -180 + 579.6$$
$$= 399.6 \text{ ft.}$$

EXERCISE III.

1. If a body fall from rest under the action of gravity find

(1) Space described in 10 seconds.

(2) Velocity acquired in 5th second.

(3) Time in falling 161 ft.

(4) Space passed through in 8th second.

(5) Space described in acquiring a velocity of 75 ft. per second.

(6) Velocity acquired in 12 seconds.

(7) Time in acquiring a velocity of 322 ft.

(8) Velocity acquired in falling 805 ft.

2. A body is projected upwards with a velocity of $4g$ ft.; how high will it rise?

3. A body is projected upwards with a velocity of 60 ft. per sec.; when and how far will it fall?

4. What must be the initial velocity upwards in order that the body may rise 120 ft.?

5. Two balls are dropped from the top of a tower, one of them 2 secs. after the other; how far will they be apart 3 secs. after the last was let fall?

6. With what velocity must a body be projected upwards that it may describe 300 ft. in 4 secs.?

7. With what velocity must a body be projected upwards so as to return to starting point in 10 secs.?

8. Three secs. after a body is let fall, another is thrown down, with a velocity of 180 ft. per sec.; when will it overtake the former?

9. A ball is projected upwards with a certain velocity, and at the same instant another ball is let fall from the height to which the first will rise; when and where will the two balls pass each other?

10. A stone let fall from a balloon ascending at the rate of 400 ft. per minute, reaches the ground in 10 secs.; find the height of the balloon, when the stone was dropped and when it reached the ground.

11. In question 10, if the balloon was descending, find its height when stone was dropped.

12. A man standing on a platform which descends with a uniform velocity of 2 ft. per sec., drops a stone which reaches the bottom in 8 secs.; how far did it fall?

13. A man standing on a platform which descends with a uniform acceleration of 10 ft. per sec., having descended for 4 secs., drops a stone which reaches the bottom in 6 secs.; what will be its terminal velocity, and how far did it fall?

14. In question 13, if the man threw the stone upwards with a velocity of *(a)* 20 ft. per sec.; *(b)* 60 ft. per sec.; in what time will the stone reach the man?

15. A stone is thrown upwards with a velocity of 161 ft. per sec., and 2 secs. afterwards another stone with a velocity of 225·4 ft. per sec ; when and where will the stones meet?

16. In question 15, if the stones had been projected downwards, when and where would they have met?

17. A stone is thrown upwards with a velocity of 100 ft. per sec., and 3 secs. afterwards another stone is thrown upwards with a velocity of 200 ft. per sec.; find their distance apart, 4 secs. after the first stone started.

18. A stone is thrown downwards from the top of a tower 3 secs. after one is dropped ; with what velocity must it be thrown so as to overtake the other in 4 secs.?

19. How high will a body rise which is thrown vertically upwards, and returns to the hand in 10 secs.?

20. A stone falling for 2 secs. breaks a pane of glass, and thereby loses one-half of its velocity ; find the space described in 6 secs. from starting.

21. A body is projected upwards with a velocity of 200 ft. per sec.; when and at what height will its velocity be 80 ft. per sec.?

22. A stone falling for 4 secs. passes through a pane of glass, thereby losing one-third of its velocity, and reaches the ground 3 secs. afterwards; find the height of the glass.

23. Gravity at the surface of the planet Jupiter being about 2·6 times as great as at the surface of the earth, find the distance and velocity acquired by a body falling for 5 secs. towards Jupiter.

24. A stone is projected upwards with an initial force of 60 ft. per sec., and an accelerating velocity of 10 ft. per sec.; how high will it rise?

CHAPTER IV.

COMPOSITION OF VELOCITIES.

11. The **resultant velocity** of two or more forces moving in the same straight line is the algebraic sum of their velocities.

12. **Composition of Velocity not in the same straight line.**—It has been determined by experiment that if two forces not in the same straight line act upon a body so as to move it, it will move in the direction of the diagonal of the parallelogram formed by the lines denoting the forces; and that this diagonal will denote the resultant velocity when the sides of the parallelogram denotes the velocities of the forces respectively.

This theory is called the Parallelogram of Velocities, and may be stated thus:—*If a body tend to move with two uniform velocities, represented by the two sides of a parallelogram drawn through a fixed point, then the resultant velocity will be represented by the diagonal of this parallelogram that passes through the same point.*

13. In order that three velocities may neutralize one another, the third must be equal to, and act in a contrary direction to the resultant of the other two. Therefore the three velocities that neutralize one another can be represented by the three sides of a triangle taken in order.

14. From the foregoing we see that, "*When several forces act simultaneously on a body, each produces the same effect as if it had acted separately.*"

15. **Resolution of Velocities** is finding the component velocities. Since the diagonal of a parallelogram represents the resultant of two component velocities, therefore we may resolve any velocity into two components by forming a parallelogram having a diagonal equal to the resultant velocity, when the sides of the parallelogram will represent the components.

EXERCISE IV.

(NOTE—Motion in the following questions is supposed to be rectilineal.)

1. A body tends to move with velocities of 10 feet and 24 feet per sec. along two straight lines at right angles to each other; find the resultant velocity.

2. A body is simultaneously urged to move with velocities of 15 ft., 18 ft., and 40 ft. per sec. respectively; can the body remain at rest?

3. A body is simultaneously urged to move with three uniform velocities, one of which would cause it to move 12 ft. east in 3 secs., another 8 ft. in 1 sec. in the same direction, and the third 24 ft. west in 4 secs. Where will the body be in 10 secs.?

4. A body falling with a uniform velocity of 20 ft. per sec. is urged horizontally with a uniform velocity of 16 ft. per sec.; find its distance from starting point after 4 secs.

5. In question 4, if the horizontal force was an acceleration of 6 ft. per sec.; find space described by the body.

6. A stone is dropped from the top of the mast of a ship which is sailing at the rate of 20 miles per hour; if the mast is 48.3 feet high, find the distance traversed by the stone, and ship, before the stone reaches the vessel.

7. A ball thrown horizontally from the top of a tower with a velocity of 30 ft. per sec. strikes the ground after 4 secs.; find the height of the tower, and how far from the base of the tower the ball struck the ground.

8. A ball whilst falling from the top of a tower is attracted horizontally by an accelerating force of 4 ft. per sec.; find the distance described by the ball in 5 secs.

9. In question 8, if the tower was 60 ft. high with what velocity would the ball strike the ground?

10. A body is projected upwards with a velocity of 64 ft. per sec., and at the same time horizontally with a velocity of 30 ft. per sec.; find its greatest height and its time of flight ($g = 32$).

11. In question 10, if the horizontal velocity was an acceleration of 20 ft. per sec.; find its greatest height, range, and time of flight.

12. If a hole were bored in the earth towards its centre, and a stone dropped down the centre of the hole, would it continue to fall down the centre of the hole or not, and why?

13. A body is moving in a certain direction at the rate of 60 ft. per sec.; find the components of its velocity along lines inclined to its direction at angles of (*a*) 30°, (*b*) 45°, (*c*) 60°, and (*d*) 120° respectively.

14. A body tends to move with a uniform velocity of 20 ft. per sec. in a certain direction, but is constrained

to move in a direction inclined at an angle of (*a*) 30°, (*b*) 45°, (*c*) 60°, to the original direction; find the component of its velocity in the latter directions.

15. In question 14, if the velocity was an acceleration of 30 ft. per sec., find components of its acceleration.

16. A body is acted on by two velocities of 15 ft. each per sec. inclined to each other at an angle of 120°; find the resultant velocity.

17. What acceleration along a certain line is equivalent to an acceleration of 60 ft. per sec. in a direction that makes an angle of (*a*) 30°, (*b*) 45°, (*c*) 60°, with that line?

18. A shot is fired in a direction inclined at angles of (*a*) 30°, (*b*) 45°, (*c*) 60°, to the horizon with a velocity of 400 ft. per sec.; find its greatest height, horizontal range, and time of flight.

19. A body is acted on simultaneously by three uniform velocities of 12, 18, and 20 ft. per sec. respectively; find the resultant velocity if the angle between first and second is 15°, and angle between second and third is 30°.

20. In question 19, if the three velocities were accelerations of 6, 8, and 12 ft. per sec., find resultant velocity in 10 secs.

CHAPTER V.

MOTION ON AN INCLINED PLANE.

16. The acceleration of a body down a plane may be regarded as the resultant of two velocities, the acceleration of gravity which acts vertically downwards, and the uniform velocity of resistance of the plane which acts at right angles to the plane.

Therefore if f denotes the acceleration down the plane and g the acceleration of gravity; by the resolution of velocities we have

$$\frac{f}{g} = \frac{h}{l}.$$

If the angle of the plane is

$$30^\circ, \frac{h}{l} = \frac{1}{2}, \quad \therefore \frac{f}{g} = \frac{1}{2}.$$

$$45^\circ, \frac{h}{l} = \frac{1}{\sqrt{2}}, \quad \therefore \frac{f}{g} \doteq \frac{1}{\sqrt{2}}.$$

$$60^\circ, \frac{h}{l} = \frac{\sqrt{3}}{2}, \quad \therefore \frac{f}{g} = \frac{\sqrt{3}}{2}.$$

If the body be projected up the plane the retardation will be the same as the acceleration in falling the same distance down the plane.

$$\therefore \text{retardation} = \frac{h}{l} g.$$

17. We can find the velocity acquired, and time occupied by a body falling down or projected down a plane, by substituting the value found for f, viz., $f = \frac{h}{l} g$, in the general equations.

$$v = ft$$

$$v = V \pm ft$$

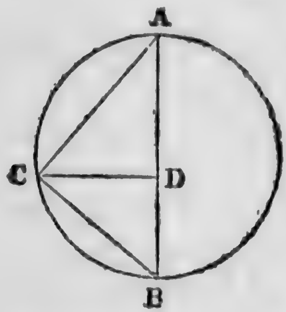
$$v^2 = V^2 \pm 2fS$$

$$v^2 = 2fS$$

$$S = \frac{ft^2}{2}$$

$$S = Vt \pm \frac{ft^2}{2}.$$

18. To find the time of falling down a chord of a circle from its highest point.



Let AB be a vertical diameter of any circle of which A is the highest point, draw any chord AC. Join CB and draw CD horizontal to meet AB in D. The acceleration down AC $= \frac{AD}{AC}g = \frac{AC}{AB}g$. (Euc. VI. 8.)

Substituting this value of f in equation $S = \frac{ft^2}{2}$, we have $S = \frac{t^2.AC}{2AB}g$, and $S = AC$, $\therefore t^2 = \frac{2.AB}{g}$, which is a constant quantity and the same time as that occupied in falling down the diameter.

Hence to find the line of quickest descent from a point to a straight line or curve, draw a circle the highest point of which shall be the given point, and which shall touch the given line or curve.

EXERCISE V.

1. Find the time occupied in falling down an inclined plane whose length is 30 ft. and height 10 ft.

2. Find velocity acquired by a body falling down an inclined plane whose height is 20 ft. and length 80 ft.

3. Find velocity acquired in 10 seconds by a body falling down an inclined plane rising 2 ft. in 5 ft.

4. The angle of a plane is 30° and length 40 ft.; find the time required for a body to fall down the plane.

5. In question 4, find velocity with which a body must be projected up the plane to reach the top.

6. A body is projected up a plane whose inclination is 45°, with a velocity of 24 ft. per second; find the space described in 3 seconds.

7. Find what initial velocity must be given to a body falling down a plane rising 1 ft. in 50 ft. so as to reach the bottom in $1\frac{2}{3}$ seconds, with a velocity of 20 miles an hour.

8. Find the space traversed in 3 minutes by a steam engine moving with an initial velocity of 10 miles an hour up a grade rising 1 in 300, neglecting friction.

9. A body is projected down a plane with a velocity of 20 ft. per second; find the velocity acquired in 4 seconds, if the inclination is 60°.

10. A ball on an inclined plane rising 1 in 200 is projected up the plane with a velocity of 32.2 ft. per second, where will it be in 7 minutes after starting?

CHAPTER VI.

ENERGY—WORK.

19. The energy of a body is its power of doing work or overcoming resistance. There are two kinds of energy, viz., Kinetic and Potential.

20. Kinetic energy is the name given to the energy of moving matter. Potential energy is the name given to the tendency of a body at rest to produce motion, *i. e.* it is the latent or stored up energy.

Suppose a moving mass whose weight is W has a velocity of v and suppose S to be the vertical height to which W would rise if projected upwards with the velocity v, then WS is the measure of the work which W is capable of doing; and

$$\because v^2 = 2gS,$$

$$S = \frac{v^2}{2g},$$

$$\text{and } WS = \frac{W\ v^2}{2g}.$$

Since weight is the tendency of a body at rest to move towards the centre of the earth and this tendency is produced by gravity acting on its mass,

$$\therefore W = Mg.$$

Substituting this value of W in $WS = \frac{W v^2}{2g}$,

$$\text{we have } W\ S = \frac{M\ v^2}{2},$$

i. e., the kinetic energy of a mass M moving with a velocity v is equal to $\frac{M\ v^2}{2}$.

21. **Relation between Force, Momentum, and Energy.**—The product of the accelerating force and the mass is called the moving force.

Then by the third law of motion, taking P for force, M for mass,

$$f = \frac{P}{M} \text{ or } P = Mf.$$

And if the moving body has a velocity v which will cause it to move for t secs. against P, then

$$P = \frac{M\,v}{t} \text{ or } Pt = Mv.$$

$$\text{Again} \because v^2 = 2fS,$$

$$\therefore f = \frac{v^2}{2S},$$

$$\therefore Mf \text{ or } P = \frac{M\,v^2}{2S},$$

$$i.\,e.,\ PS = \frac{M\,v^2}{2} = WS = \text{energy}.$$

$$\text{Substituting } M = \frac{W}{g} \text{ in } P = Mf,$$

$$\text{we have } P = \frac{Wf}{g} \text{ or } Pg = Wf.$$

Hence the momentum is equal to the force multiplied by the time of motion. The energy of a moving body is equal to the force multiplied by the space traversed. By taking the force, time and space to be unity in above equations, we see that the *unit* of *momentum* is that momentum produced by a unit of force acting for unit of time, and the *unit* of *energy* is the energy produced by a unit of force acting through unit of space.

EXERCISE VI.

1. How much work is done per hour if 200 lbs. be raised 3 ft. in 2 minutes?

2. A train weighing 20 tons moves for half an hour at the rate of 25 miles an hour. How much work is done, the total resistance being 10 lbs. per ton?

3. A train weighing 10 tons having an initial velocity of 10 miles per hour receives an acceleration of 5 miles per hour for 4 hours; find the work done in the given time, if total resistance is 8 lbs. per ton.

4. How much energy will be expended in excavating a pit 10 ft. deep, having an area of 6 square feet, supposing a cubic foot of earth to weigh 100 lbs.?

5. A body weighing 60 lbs. is thrown vertically upwards with a velocity of 100 ft. per sec.; find the energy expended when the body is at its highest point.

6. Find the energy acquired by a body weighing 100 lbs. falling under the force of gravity for 10 seconds.

7. Find the energy expended in raising a weight of 2 tons with a velocity of 10 ft. per sec. up an inclined plane rising 1 foot in 50 ft., during 15 minutes.

8. A body weighing 500 lbs. is moving with a velocity of 80 ft. per sec., and afterwards it is found to move with a velocity of 40 ft. per sec.; find amount of work done.

9. The velocity of a moving body whose weight is W is reduced from V to v; find energy lost.

10. For how long a time must a force of 4 lbs. act on a mass whose weight is 20 lbs. to generate a velocity of 80 ft. per sec.?

11. A force of 3 lbs. moves a certain mass from rest, through 20 ft. in 2 secs.; find the weight of the mass.

12. A plane supporting a weight of 20 lbs. is falling with an acceleration of 12 ft. per sec.; find the pressure on the plane.

13. Through what distance must a force of 1 lb. act upon a mass of 64.4 lbs. in order to increase the velocity from 20 ft. to 30 ft. per sec.?

CHAPTER VII.

FRICTION.

22. The resistance which a force has to overcome in moving one body upon another is termed friction.

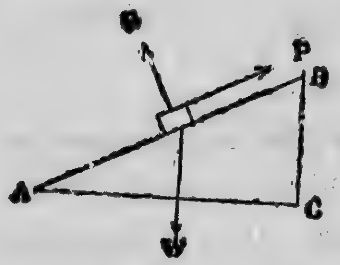
23. **Measure of Friction.**—It is evident that the resistance of friction always acts in the opposite direction to that in which the body tends to move. Friction may be measured experimentally by finding the force necessary to move a body on a horizontal plane.

Place a body whose weight is W on a horizontal surface AB. Attach a fine cord to W and let it pass over a pulley at B, and support a weight P. If P is the least weight that will move W along the plane, P is the measure of friction between the two surfaces. By changing the materials of the plane AB and W, we can find the friction between any two bodies.

The co-efficient of friction is the relation of the power P to the weight W (or reaction of the plane), *i. e.* taking μ for co-efficient of friction

$$\mu = \frac{P}{\text{Reaction of plane}}.$$

Another method of measuring friction is by elevating the plane AB until the body begins to slide down the plane.



Let a weight W rest upon the inclined plane BAC, and denote the reaction of the plane by R, and resistance due to friction by force P, which is the force acting parallel to the plane that would support the weight W.

From theory of inclined plane,

$$\frac{P}{R} = \frac{h}{b}, \quad \therefore \text{ co-efficient of friction,}$$

$$\text{or } \mu = \frac{h}{b}.$$

24. **Laws of Friction.**—By the foregoing means the following laws have been experimentally established :—

(1) The friction is independent of the extent of surfaces in contact.

(2) The friction varies directly as the weight or pressure applied, the surface in contact remaining the same.

(3) The friction is independent of the velocity when there is motion.

EXAMPLES.

1. A body is just on the point of sliding on a rough plane that rises 5 ft. in 13 ft.; find the co-efficient of friction.

$$\mu = \frac{h}{b} = \frac{5}{12},$$

i. e. the friction is $\frac{5}{12}$ of the pressure of the body on the plane.

2. Find tension of a string just moving a body weighing 12 lbs. up an inclined plane rising 7 ft. in 25 ft.; the co-efficient of friction being $\frac{1}{4}$.

The friction always acting against motion acts against tension of the string, $\therefore$ force acting up the plane is $\tau - \frac{1}{4}\mathrm{R}$.

$$\therefore \frac{\tau - \frac{1}{4}\mathrm{R}}{\mathrm{W}} = \frac{h}{l} = \frac{7}{25},$$

$$\text{and } \frac{\mathrm{R}}{\mathrm{W}} = \frac{b}{l} = \frac{24}{25}, \quad \therefore \mathrm{R} = \frac{24}{25}\mathrm{W}.$$

$$\therefore \frac{\tau - \frac{6}{25}\mathrm{W}}{\mathrm{W}} = \frac{7}{25}, \text{ and } \mathrm{W} = 12,$$

$$\tau - \frac{72}{25} = 12 . \frac{7}{25},$$

$$\tau = \frac{72 + 84}{25} = \frac{156}{25} = 6\frac{6}{25} \text{ lbs.}$$

EXERCISE VII.

1. A body is just on the point of sliding down a rough plane which rises 8 ft. in 17 ft.; find co-efficient of friction.

2. A body weighing 30 lbs. is just supported by friction on a plane rising 9 ft. in 41 ft.; find μ.

3. A body weighing 60 lbs. is just supported by friction on a plane whose inclination is (*a*) 30°, (*b*) 45°, (*c*) 60°; find co-efficient of friction, and force exerted by it.

4. Find the energy expended in pulling a body weighing 200 lbs. 80 ft. up a plane that rises 11 ft. in 61 ft., the force of friction being 2 oz. per lb.

5. Find the energy expended in pulling a body weighing 500 lbs. for 6 seconds up a plane rising 13 ft. in 85 ft., with a uniform velocity of 10 ft. per second; μ being $\frac{1}{8}$.

6. A body weighing 40 lbs. will just rest on a plane rising 5 in 13; if the plane be tilted up so that it rises 3 in 5; find the force acting parallel to the plane necessary to support the weight.

7. A body weighing 60 lbs. is projected along a horizontal plane with a velocity of 200 ft. per second; if the co-efficient of friction is $\frac{1}{10}$; find the work done against friction in 10 seconds.

8. A body weighing 50 lbs. falls down an inclined plane whose height is 40 ft. and length 104 ft.; find the energy acquired by the body, μ being $\frac{1}{6}$.

9. A body weighing 90 lbs. is projected up a plane rising 15 in 113 with a velocity of 161 ft. per second; find its velocity when it returns to the point whence it was projected, μ being $\frac{1}{8}$.

10. A body weighing 10 lbs. projected with a velocity of 48.3 ft. per second along a horizontal plane passes over 144 ft. before coming to rest; find the resistance due to friction.

MISCELLANEOUS QUESTIONS.

(Taken from various Examination Papers.)

1. State the rule for the composition of velocities and give an instance shewing the truth of the rule.

2. Explain how a body can be made to describe the sides of a regular polygon with constant velocity by having a certain velocity impressed on it at each angular point; and calculate the magnitude of the velocity impressed on a body at each angular point of an octagon which the body describes with a constant velocity of 2 ft. per second.

3. State Newton's laws of motion, and explain the bearing of the second law upon the definition of force.

4. Deduce the theory of the composition of forces from the 2nd law of motion.

5. How are forces that produce motion measured?

6. How can the formulæ $Mv = ft$, $2Ms = ft^2$ be experimentally verified?

7. Define energy. How is it measured? Explain relation between force, energy and momentum.

8. A body moves under the action of a constant force, prove (*a*) that its velocity will be uniformly accelerated, (*b*) that the spaces described from rest will vary as the squares of the times of describing them.

9. Given an acceleration and velocity with certain units of space and time, show how to express them when the units of space and time are changed.

10. If a particle move from rest under the influence of two given uniform accelerations, making an angle of θ with one another obtain expressions for the position of the particle and for the resultant velocity at the end of any time.

11. A certain acceleration is represented by 32 when one foot and one second are the units of space and time. By what number will it be represented when 4 ins. and 3 secs. are the units?

12. Obtain a formula for finding the space described in a given time by a body moving with a given uniform velocity.

13. Find work done in the following cases :—(*a*) Raising a body of given weight to a given height along a rough inclined plane (co-efficient of friction μ), whose inclination is known. (*b*) Raising a window-blind of given dimensions and weight by means of a roller at its top.

14. What is uniformly accelerated, and what uniformly retarded motion? Give examples of each. What are the relations existing among the quantities denoting velocity, space, force and time in such motion?

15. A heavy cylinder would take longer to roll down an inclined plane than to slide down it without friction. Explain why.

16. If a body be projected upwards show what changes take place in the character of its energy from the time it leaves the earth's surface until it returns to it again.

17. Explain how a variable velocity is measured and how that measure is expressed.

18. The velocity of a body falling freely receives each second an acceleration of 32 ft. per sec. Express this

acceleration taking the mile the unit of length and the hour as the unit of time.

19. Define the absolute or kinetic, and the gravitating or static units of force, and state approximately the ratio they bear to each other.

20. A heavy particle is projected in vacuo with a given velocity. Determine its position and the magnitude and direction of its velocity after a given time.

21. A shot of 2,000 lbs. is fired from a gun weighing 224,000 lbs. placed on a smooth horizontal plane, and elevated at an angle of 30°. Find the horizontal range of the shot (neglecting the resistance of the air), if its nozzle velocity relative to the gun be 1,500 ft. per sec.

22. A railway train is moving at the rate of 30 miles per hour, and a person in one of the carriages tosses up a ball vertically with respect to the carriage, with a velocity of 12 ft. per sec. Determine the path of the ball with respect to the ground.

23. The drops of a shower are falling straight down, but to a person sitting in a railway carriage moving at the rate of $22\frac{1}{2}$ miles per hour they appear to fall at an angle of 30° from the vertical. At what rate in feet per sec. are the drops falling?

24. If a body in motion be acted on by a constant force in its line of motion, what will be the effect on the motion during the time the force continues to act? What would be the effect on the motion if the force were to suddenly cease acting?

25. A body thrown vertically upwards returns to the earth again in 5 secs. How high did it rise and what was its initial velocity? Had the body been thrown at an

angle of 60° elevation and returned to the earth again in 5 secs., find its greatest height and initial velocity.

26. Two weights whose masses are 10 and 6 respectively are connected by a weightless string passing over a smooth pulley. Find the kinetic energy of the system 5 secs. after the beginning of the motion and obtain therefrom the acceleration of the common centre of gravitation of the weights.

27. Deduce the Parallelogram of Forces from the Parallelogram of Velocities and the Laws of Motion.

28. At one of the angles of a regular hexagon forces are applied acting towards the other angles and proportional to the distances of these angles from the point of application. Determine the magnitude of the resultant.

29. Deduce the Principle of Moments from the Parallelogram of Forces.

30. A gun (weight 3 tons) rests on a plane of inclination 30° to the horizon, being pointed downwards parallel to the plane; a shot of 60 lbs. is discharged from it with a velocity of 1,500 ft. per sec. Find how far up the plane the gun will recoil.

31. A body descending vertically draws an equal body 25 ft. in $2\frac{1}{2}$ secs. up a smooth plane inclined 30° to the horizon by means of a string passing over a smooth pulley at the top of a plane. Determine the force of gravity.

32. An iron ball weighing 10 lbs. falls from a height of 6 ft. on to a floor which yields $\frac{1}{10}$ of an inch. Assuming the ball to be incompressible determine the mean value of the force called into play during the compression of the floor.

33. A body weighing 12 lbs. slides with a uniform velocity down a plane that rises 5 in 13. How much energy would have to be expended in order to drag the body 13 ft. up the plane by means of a string stretched parallel to the plane?

34. Explain why it is dangerous to jump out of a railway carriage in motion.

35. If a person is walking in a straight line, in what direction must he throw a ball upwards that it may return into his hands?

36. Is there any difference in the velocity which a falling body acquires, when dropped from a certain height near the equator and from the same height near the poles?

37. On what does the weight of a body depend?

38. Which could you throw further, a small ball of lead or a ball of cork of the same size? Why?

39. At the earth's equator the hot air ascends, and is replaced by cold air which blows along the ground from the poles. That which comes from our hemisphere blows from the north-east instead of from the north. Explain this.

40. Distinguish between weight and mass. Under what circumstances will the weight of a body vary whilst its mass remains the same?

ANSWERS TO STATICS.

EXERCISE I.

2. (*a*) $10\frac{1}{2}$ inches. (*b*) 21 inches. (*c*) $31\frac{1}{2}$ inches.
3. (*a*) 20 lbs. (*b*) 42 lbs. (*c*) 120 lbs.

EXERCISE II.

1. 14 lbs. 2. 4 lbs. in direction of 12 lbs. 3. When they act together in same straight line. When they act in opposite directions. When they are equal and opposite. 4. 14 lbs. in direction of resultant. 5. 16 lbs. 6. 5 and 10 lbs. 7. 64 lbs. in direction of 16 lbs. and 48 lbs. in opposite direction. 8. 23 lbs. 9. 50 lbs. 10. 80 lbs. 11. In 9 zero. In 10, 20 lbs. in direction of 80 lbs. 12. In opposite directions. 13. 30 lbs. and 20 lbs.

EXERCISE III.

4. 61 lbs. 5. 84 lbs. 6. If P represent the smaller force, then resultant is $\frac{17}{8}$ P, and an equal force in opposite direction will produce equilibrium. 7. 4 men pulling together. 8. 37 lbs. 9. 45 lbs. and 336 lbs. 10. 24 lbs. and 25 lbs.

EXERCISE IV.

1. 5 and $5\sqrt{3}$. 2. (*a*) $4\sqrt{3}$, $2\sqrt{3}$. (*b*) $\frac{16}{3}\sqrt{3}$, $\frac{8}{3}\sqrt{3}$. (*c*) 8, 4. (*d*) $\frac{4}{3}$, $\frac{2}{3}$. 3. (*a*) $2\sqrt{39}$ ft. (*b*) $\sqrt{91}$ ft. 4. $6\sqrt{2}$ ft. 5. 23.38 ft. 6. (*a*) $6.47a$ ft. (*b*) $2.05a$ ft. (*c*) $2.83a$ ft. 7. ABC 90°, ACB, 30°, BAC, 60°.

EXERCISE V.

1. $50\sqrt{3}$. 2. 28 lbs. 3. 17.3 lbs. 4. 11.08 lbs. 5. 13.11 lbs. 7. 35 lbs. 8. $20\sqrt{2}$ lbs. 9. 30°. 10. 45°. 11. 10 lbs. 12. $24\sqrt{3}$ lbs. 13. 31.06 lbs. 14. 26.1 lbs. 15. 29.7 lbs. and 44.6 lbs. 16. 15 lbs.

EXERCISE VI.

5. Yes. 8. 12 lbs. 9. 2 lbs. in direction of 4 lbs. 10. At right angles. 11. 32 lbs. and 60 lbs. 13. $2\sqrt{2} : 3$. 15. 120°. 16. 61P. 18. AD. 19. 200 lbs. 20. 26 lbs. and 168 lbs. 21. 30 lbs. 22. 60 lbs. and 45 lbs. 23. (*a*) 74.66 lbs. and 7.12 lbs. (*b*) 100 lbs. (*c*) $8\frac{1}{3}$ lbs. (*d*) $\frac{3}{4}$P lbs. 24. $10\sqrt{6}$ lbs. 25. If AB, BC, CD be the 3 sides then AD is the resultant. 26. $12\sqrt{3}$ lbs. 27. 78 lbs. 28. (*a*) $22\frac{1}{2}$ lbs. (*b*) 80 lbs. 29. 60 and $60\sqrt{3}$ lbs.

30. $F = 100\sqrt{3}$ lbs.; $R = 100$ lbs.

31. (*a*) $F = R = 100\sqrt{2}$ lbs. (*b*) $F = 100$ lbs.; $R = 100\sqrt{3}$ lbs. 32. 12 lbs. 33. $30\sqrt{3}$ and 60 lbs.

EXERCISE VII.

4. $40\sqrt{3}$ lbs. 5. $F = 40\sqrt{3}$ lbs.; $W = 80\sqrt{3}$ lbs. 6. 240 lbs. each. 7. (*a*) 120 lbs. (*b*) $120\sqrt{3}$ lbs. (*c*) $120\sqrt{3}$ lbs. 8. (*a*) R. of wall $50\sqrt{3}$; R. of hinge $150\sqrt{\frac{13}{3}}$. (*b*) 150 lbs.; and R. of hinge $150\sqrt{5}$ lbs. (*c*) $150\sqrt{3}$ lbs.; and $150\sqrt{7}$ lbs. 9. (*a*) $15\sqrt{5}$ lbs.; and 15 lbs. (*b*) $30\sqrt{2}$ lbs.; and 30 lbs.; (*c*) $30\sqrt{5}$ lbs.; and 60 lbs. 10. $\dfrac{W\sqrt{2rh - h^2}}{r - h}$.

EXERCISE VIII.

6. $40\sqrt{2}$ lbs. 7. 168 lbs. 8. $\frac{P}{\sqrt{10}}$, and $\frac{3P}{\sqrt{10}}$.

9. $\frac{50}{\sqrt{2+\sqrt{3}}}$ lbs. 10. $50\sqrt{3}$ lbs. 11. $\frac{200}{\sqrt{2-\sqrt{2}}}$ lbs.

12. $\frac{60}{\sqrt{2+\sqrt{2}}}$ lbs. 13. 80 lbs. 14. $\frac{120}{\sqrt{2-\sqrt{3}}}$ lbs.

15. 144 lbs. 16. 25 lbs., and $25\sqrt{3}$ lbs. 17. 36 lbs. 18. $9\sqrt{5}$ lbs. 19. $\sqrt{3}$ times one of the fcs. 20. $\sqrt{2-\sqrt{2}}$ times one of the forces. 21. 17.7. 22. 1.98 times one of the forces. 23. $40\sqrt{3}$ lbs. 24. 45.27 lbs. 25. 37.68 lbs. 26. 60 lbs. and $60\sqrt{3}$ lbs. (*b*) $60\sqrt{2}$ each. (*c*) $60\sqrt{3}$; and 60 lbs. 27. $4\sqrt{2}$ lbs. 28. Let AB, AC, AD, AE, AF, represent fcs. resolve along AD and at right angles to it. 29. $4\sqrt{3}$ lbs. 30. $10\sqrt{3}$ lbs. 31. 64.6 lbs. 32. 5 lbs. 33. 10 lbs. 34. 30°. 35. (*a*) $20\sqrt{3}$ lbs. (*b*) 60 lbs. (*c*) $60\sqrt{3}$ lbs. 36. $26\frac{2}{3}$ and $53\frac{1}{3}$. 37. Fcs. are equal. 38. $\frac{15}{2}(\sqrt{3}-1)$ lbs., reaction $15\sqrt{3}$ lbs. 39. $60\sqrt{2}$ lbs.

EXERCISE IX.

3. $12\frac{1}{2}$ inches from 49 lbs. 4. 30 inches. 5. One nearest wt. 120 lbs.; other 60 lbs. 6. 35 in. from 8 lbs. 7. 8 inches. 8. 5 : 2. 9. 3 lbs. and 9 lbs. 10. 15 inches from one end. 11. At a point in a median line $\frac{2}{3}$ of its length from the vertex. 12. 5 lbs., 40 inches from 25 lbs. 13. At a point C so that AC = $\frac{3}{5}$ AB. 14. 28 inches from heavier weight. 15. $7\frac{11}{27}$ lbs. at A, $5\frac{25}{27}$ lbs. at B, $2\frac{26}{27}$ lbs. at C, $3\frac{19}{27}$ lbs. at D. 16. Let AB = 9 inches, 25 lbs. at A, 40 lbs. at B, 20 lbs.

at C, 15 lbs. at D. 17. Bisect AB, CD in E and F. take FH $\frac{5}{14}$ of EF ; H is position of resultant.

EXERCISE X.

4. 12 lbs., 5 inches from 9 lbs. 5. 5 lbs. 6. 1 foot from 4 lbs. 7. 3 inches from 10 lbs. 8. 3 inches. 9. 3 : 5. 10. $10\frac{2}{3}$ cwt. 11. $2\frac{1}{2}$ lbs. 12. (*a*) $7\frac{1}{2}$ lbs., (*b*) 5 lbs. 13. $2\frac{2}{11}$ ft. from 15 lbs.; $24\frac{3}{4}$ lbs.

EXERCISE XI.

2. $68\frac{4}{7}$ lbs. 3. Reaction of wall $45\frac{5}{7}$ lbs., of floor 400 lbs. 4. (*a*) $60\sqrt{3}$ lbs., (*b*) 60 lbs., (*c*) $20\sqrt{3}$ lbs. 5. (*a*) $90\sqrt{3}$ lbs. and $\frac{255}{2}\sqrt{3}$ lbs., (*b*) 90 lbs. and $127\frac{1}{2}$ lbs., (*c*) $30\sqrt{3}$ lbs. and $\frac{85}{2}\sqrt{3}$ lbs. 6. (*a*) $80\sqrt{3}$ lbs., (*b*) 80 lbs., (*c*) $\dfrac{80}{\sqrt{3}}$ lbs. 7. $\frac{3}{4}$ of its length from the foot. 8. (*a*) pressure $= \frac{165}{16}\sqrt{3}$ lbs., R $= \frac{165}{8}\sqrt{3}$ lbs. (*b*) $\frac{165}{8}$ lbs., R $= \frac{165}{8}\sqrt{2}$ lbs. (*c*) P $= \dfrac{165\sqrt{3}}{16}$, R $= \frac{165}{8}$.

EXERCISE XII.

7. 7 inches from 16 lbs. 8. $1\frac{3}{7}$ ft. from 20 lbs. 9. $2\frac{3}{5}$ ft. from 12 lbs. 10. $12\frac{3}{11}$ inches from 30 lbs. 11. $9\frac{1}{7}$ inches from A in perpendicular from A on BC. 12. $9\frac{1}{3}$ inches from A. 13. 24 lbs. 14. $13\frac{1}{2}$ inches from 12 lbs. 15. $10\frac{2}{15}$ in. from 15 lbs. 16. 1 ft. 2 in. 18. Centre of gravity of triangle. 19. Let AB, BC, CD be sides of square, bisect them in E, F, H, join EH, and bisect it in K ; join FK ; take KG $\frac{1}{3}$ KF, G is point required. 20. $\dfrac{2h_1 + h_2}{3}$ from vertex of triangle whose height is h_1. 21. It may easily be proved that centre of

gravity lies in line joining 6 and 9 lbs., also in line joining 11 and 8 lbs., $\therefore$ centre of gravity must be at centre of hexagon. 22. $60(\sqrt{2}+1)$. 23. 60 lbs. 24. 4 ft. from right angle in line joining right angle with middle of hypothenuse. 25. $2\frac{5}{7}$ inches from heavy end. 26. $1\frac{10}{11}$ inches. 27. Centre of square. 28. $\frac{\sqrt{2}}{15}$ ft. 29. $2\sqrt{2}$ ft. from 18 lbs. in line joining 18 and middle of square. 30. 144 ft. 31. (*a*) $8\sqrt{3}$ inches. (*b*) 8 inches. (*c*) $\frac{8}{\sqrt{3}}$ inches. 32. (*a*) $6\sqrt{3}$ ft. (*b*) 6 ft. (*c*) $2\sqrt{3}$ ft. 33. In common axis where two cylinders join. 34. $17\frac{16}{17}$ ft. from top. 35. $3\frac{3}{7}$ inches from bottom. 36. 22 inches. 37. $\frac{9}{22}\sqrt{2}$ inches from geometrical centre of square. 38. $\frac{5}{7}$ inches from geometrical centre. 39. $\frac{16}{21}\sqrt{2}$ inches from geometrical centre. 40. $\frac{2}{3}$ inches from centre of plate. 41. $\frac{5}{6}$ in. from centre of disc. 42. $\frac{ad^2}{D^2-d^2}$ from centre of disc. 43. 4 inches from geometrical centre.

EXERCISE XIII.

1. 20 inches from fulcrum. 2. 32 lbs. 3. $5\frac{1}{7}$ inches from 50 lbs. 4. 20 inches; 9 lbs. 5. $26\frac{2}{3}$ lbs. 6. $18\frac{1}{3}$ lbs. 7. 40 lbs.; $20\sqrt{7}$ lbs.

EXERCISE XIV.

2. 5 lbs. 3. $4\frac{4}{7}$ lbs.; $3\frac{1}{2}$ lbs. 4. $\frac{2}{3}$ of an inch from fulcrum. 5. $3\frac{5}{7}$ inches from 12 lbs. 6. $10\frac{2}{3}$ lbs. 7. 60 cents ℔ lb. 8. $1\frac{2}{3}$ % gain. 9. 7 lbs.; $13\frac{1}{13}$ ins. and $16\frac{2}{13}$ ins. 10. 11 lbs. 11. $55\frac{5}{9}$ %.

EXERCISE XV.

1. 72 lbs. 2. 12 lbs. 3. 270 lbs. 4. 2160 lbs.
5. 1200 lbs. 6. 180 lbs. 7. $14\frac{2}{5}$ lbs.

EXERCISE XVI.

3. 400 lbs. 4. 5. 5. 70 lbs. 6. $46\frac{2}{3}$ lbs.
7. $17\frac{8}{9}$ lbs. 8. 4 lbs. 9, 320 lbs.

EXERCISE XVII.

1. 16 lbs. 2. $16\frac{1}{2}$ lbs. 3. 20 lbs. 6. 20 lbs.
7. 64 lbs.

EXERCISE XVIII.

1. 75 lbs. 2. 5. 3. 2 lbs. 4. 24 lbs.
5. 430 lbs. 6. 5 lbs. 7. $1\frac{1}{2}$ lbs.

EXERCISE XIX.

2. $P = 25$; $R = 60$. 3. $W = 50$; $R = 48$.
4. $R = 80$; $l = 41$. 5. $P = 33$; $W = 183$; $R = 180$.
6. $P = 13$; $W = 85$; $R = 84$. 7. $30°$. 8. $18\sqrt{3}$ lbs.
9. 30 lbs. 10. 64 lbs. and 136 lbs. 11. 3 : 11.
12. 5 : 13. 13. $5\sqrt{10}$ lbs. 14. $30°$. 15. 32 lbs.

EXERCISE XX.

1. 48 lbs. 2. $b = 15$; $l = 17$; $R = 34$.
3. $P = 24$; $R = 74$. 4. $R = 25$; $h : b : l :: 7 : 24 : 25$.
5. $P = 32$; $b = 63$; $l = 65$. 6. $W = 160$; $h = 9$; $l = 41$.
7. $P = 39$; $W = 252$; $R = 255$. 8. $87\frac{33}{41}$. 9. $45°$.
10. $\frac{40}{\sqrt{3}}$ lbs. 11. $60°$. 12. $79\frac{3}{4}$ lbs. 13. $6\sqrt{3}$ lbs.
14. 5 lbs. 15. 50 lbs.

EXERCISE XXI.

(Taking $\pi = 3\frac{1}{7}$.)

1. $1885\frac{5}{7}$ lbs. 2. 616. 3. $5\frac{15}{22}$ lbs. 4. $\frac{22}{25}$ inches.
5. $\frac{105}{176}$ lbs. 6. 1620 lbs. 7. $48\sqrt{3}$ lbs. 8. 440.
9. $2\frac{11}{12}$ inches.

EXERCISE XXII.

1. $1\frac{3}{5}$ ins. 2. 19 lbs. 3. 90 lbs. 4. 110 lbs.
5. $1\frac{1}{6}$ ft. 6. 5 movable. 7. 5. 8. 1 : 144. 9. 2 : 3.

ANSWERS TO DYNAMICS.

EXERCISE I.

1. 75 ft. per sec. 2. 2,700°. 3. $8\frac{1}{2}$°. 4. 1.1 ins.
5. $36\frac{2}{3}$ ft. per sec. 6. 20 yds. from A. 7. In $26\frac{2}{3}$ min.
8. No.

EXERCISE II.

1. 480 ft. 2. 1,000 ft. 3. 10 ft. per sec.
4. 10 per sec. 5. 8 secs. 6. 360 ft. 7. 6 secs.
8. 120 ft. per sec. 9. 32 ft. per sec. 10. 72 ft. per sec.
11. 72 ft. per sec. 12. 24 ft. per sec. 13. 180 ft.

EXERCISE III.

1. (1) 1,610 ft.; (2) 161 ft. per sec.; (3) $\sqrt{10}$ secs.
(4) $24\frac{1}{2}$ ft.; (5) 87.3 ft.; (6) 386.4 ft. per sec.; (7) 10 secs.;
(8) 161 $\sqrt{2}$ ft. per sec. 2. 8g ft. 3. 3.7 secs., 59 ft.
4. .88 ft. per sec. 5. 257.6 ft. 6. 139.4 ft. per sec.
7. 161 ft. per sec. 8. 1.73 secs. 9. $\frac{3}{4}$ of way up.
10. (*a*) $1543\frac{1}{3}$ ft.; (*b*) 1610 ft. 11. $1676\frac{2}{3}$ ft. 12. 1046.4 ft.
13. 233.2 ft. per sec.; 819.6 ft. 14. (*a*) 2.7 secs.;
(*b*) 5.405 secs. 15. In 2 secs. more; 386.4 ft. high.
16. They would never meet. 17. 441.5 ft.
18. 132.8 ft. per sec. 19. $402\frac{1}{2}$ ft. 20. 450.8 ft.
21. 3.72 secs.; 521.73 ft. 22. $412\frac{1}{2}$ ft. 23. $1046\frac{1}{3}$ ft.;
velocity 418.6 ft. per sec. 24. $81.0\dot{8}\dot{1}$ ft.

EXERCISE IV.

1. 26 ft. per sec. 2. No. 3. 60 ft. E. of starting point. 4. $16\sqrt{41}$. ft. 5. $16\sqrt{34}$ ft. 6. Stone 70 ft.; ship $\frac{88}{3}\sqrt{3}$ ft. 7. (1) 64.4 ft.; (2) 120 ft. 8. 405.5 ft. 9. $4\sqrt{15g}$. 10. 64 ft.; 4 secs. 11. Height 64 ft.; range 160 ft.; time 4 secs. 12. It would strike the side of hole, because the rotary motion of the earth is greatest at the surface. 13. (*a*) 30 and $30\sqrt{3}$; (*b*) $30\sqrt{2}$; (*c*) $30\sqrt{3}$ and 30; (*d*) 120 and $120\sqrt{3}$. 14. (*a*) $\frac{20}{\sqrt{3}}$; (*b*) 20. (*c*) $20\sqrt{3}$. 15. (*a*) $10\sqrt{3}$; (*b*) 30; (*c*) $30\sqrt{3}$. 16. 15 ft per sec. 17. (*a*) $30\sqrt{3}$; (*b*) $30\sqrt{2}$; (*c*) 30. 18. Height 621.1 ft.; range $\frac{40000\sqrt{3}}{g}$; time 6.2 secs.; (*b*) $\frac{40000}{g}$, $\frac{80000}{g}$; time $\frac{200\sqrt{2}}{g}$ secs; (*c*) $\frac{60000}{g}$, $\frac{40000}{g}$, $\frac{200\sqrt{3}}{g}$ secs.

19. $\sqrt{(9+6\sqrt{2})^2+(9\sqrt{3}+6\sqrt{2}+18)^2}$.

20. $\sqrt{(40+30\sqrt{2})^2+(40\sqrt{3}+30\sqrt{2}+100)^2}$.

EXERCISE V.

1. $6\sqrt{\frac{5}{g}}$. 2. $\sqrt{40g}$. 3. $2\sqrt{g}$. 4. $4\sqrt{\frac{10}{g}}$. 5. $\sqrt{40g}$. 6. $\frac{4.950}{2\sqrt{2}}g$ ft. 7. $28\frac{13}{50}$ ft. per sec. 8. 901.2 ft. 9. 131.5 ft. per sec. 10. 3896.2 ft. down the plane.

EXERCISE VI.

1. 18,000 ft. lbs. 2. 13,200,000 ft. lbs. 3. 33,792,000 ft. lbs. 4. 30,000 ft. lbs. 5. 9316.7 ft. lbs.

6. 161,000 ft. lbs. 7. 720,000 ft. lbs. 8. 37,500 ft. lbs.
9. $\frac{W}{2g}(V^2 - v^2)$. 10. $\frac{400}{g}$. 11. 9.66 lbs. 12. $12\frac{1}{2}$ lbs.
13. 500 ft.

EXERCISE VII.

1. $\frac{8}{15}$. 2. $\frac{9}{40}$. 3. $\mu = \frac{1}{\sqrt{3}}$, 1, and $\sqrt{3}$ respectively.
Force = 30, $30\sqrt{2}$, and $30\sqrt{3}$ lbs., respectively.
4. $1967\frac{13}{81}$ ft. lbs. 5. $8294\frac{2}{17}$ ft. lbs. 6. $10\frac{2}{3}$ lbs.
7. 12,000 ft. lbs. 8. 1200 ft. lbs. 9. 10.4 ft. lbs.
10. $\frac{1}{4}$ or $2\frac{1}{2}$ lbs.

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